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TECHNICAL NOTE

Prepurchase financing pool: Revealing the *IRR* problem

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ABSTRACT

Internal rate of return (*IRR*) is one of the most common and important indicators in investment analysis because it is often used by managers and practitioners as a decision-making criterion. Moreover, the *IRR* reflects the financial cost in financing decisions and it helps to answer the following question when comparing different financing alternatives: "Which loan is the cheapest?" Among the different types of loans in Brazil, there is a financial product called a prepurchase financing pool (PPFP) that is generally regarded as the best option for financing or loans. The objective of this article is to use the prepurchase financing pool to show the flaws of *IRR* in financial analysis. In particular, when *IRR* is used to evaluate the prepurchase financing pool, one finds problems of reliability regarding (i) existence, (ii) uniqueness, and (iii) economic interpretation of the rate. The results show that the prepurchase financing pool is relevant evidence that the *IRR* flaws are found in financial products.

Introduction

People and companies buy services, goods, or real estate all the time. However, cash payment is not the only alternative in these situations. A loan can be used to pay for goods. The cost of the transaction is not only the value of the objects or the goods but includes the price plus the interest. There are many types or variations of loans in the financial market. Nevertheless, the use of a singular type of loan is common in Brazil. This loan is known as a prepurchase financing pool (PPFP), which consists of a gathering of several people or companies coming together as a group for the shared purpose of buying or acquiring things, goods or property (Brazilian Central Bank [BCB] 2014a). The PPFP is similar to the Rotating Savings and Credit Association (ROSCA) discussed by Ahn et al. (2016), with emphasis on microfinance. Generally, the PPFP is arranged by a bank or a financial institution that charges fees to operate this process. In this type of operation, the total value of the underlying object for a participant of the pool is diluted through a determined term of the operation. All of the participants contribute during the term of the prepurchase financing. In every period until the expiration of the pool, each participant contributes with an installment that adds up to buy one underlying object. Using a random lottery, in each period, one participant is chosen to receive

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the object. At the end of the term of the pool, every participant will have received one underlying object. The prepurchase financing pool can have the following features: the monthly possibility of gaining credit through a lottery, the possibility of a fast track to get the underlying object through a bid, the opportunity of creating and enlarging equity, and flexibility in the use of the credit (Associação Brasileira de Administradoras de Consórcio [ABAC; Brazilian Association of Management Companies of Pre Purchasing Financing Pool] 2014).

The installment is calculated by adding the common fund to the management fee and the reserve fund. The monthly common fund is the installment that each participant pays to form the amount of the prepurchase financing pool. The management fee is the remuneration of the prepurchase financing pool manager and its service in setting up, organizing, and managing the prepurchase financing pool. The reserve fund is a fund directed to protect the pool against some situations (for example, insolvency) provided in the contract (ABAC 2014). One way to calculate the financial cost of the prepurchase financing pool is through the internal rate of return (*IRR*) method. As described by Sarper et al. (2010), this is the interest rate at which the financed value is equal to the sum of the present value of future installments. In the investment analysis, the *IRR* must be equal to the minimum attractive rate of return. However, in the case of prepurchase financing pools, (*IRR*) represents the cost of the operation or the financial cost. In the prepurchase financing pool, the use of the *IRR* generates three kinds of flaws:

1. The *IRR* may not exist.
2. The *IRR* may be multiple.
3. The financial nature of the *IRR* (return rate *vs.* borrowing rate) is not unambiguous across participants.

Results 1 and 2 occur rather frequently, considering a numerical example and two real-life examples. As for result 3, because the *IRR*s for different participants can be either the rate of return or the borrowing rate, *IRR* cannot measure the cost of the loans incorporated in a PPFP and cannot be compared between them. These kinds of flaws can be addressed by using the average internal rate of return (*AIRR*) approach, which can, in turn, correctly capture the cost of the loans: in this context, the smaller the AIRR, the cheaper the loan.

In the next section, we show the basic concepts of the *IRR* and its flaws. Then the prepurchase financing pool is presented. The relationship among the *IRR*, *AIRR*, and prepurchase financing pools is shown next. Two real-life prepurchase financing pool situations are presented in the following section. Finally, the concluding remarks and indications of future studies are given in the last section.

Internal rate of return

IRR is a widely used tool for decision making in the evaluation of a deterministic cash flow stream (Hazen 2003; Tang and Tang 2003). As Magni (2011) and Kim and Reinschmidt (2012) suggest, *IRR* is defined as the interest rate where the net present value of a stream of cash flows is equal to zero. The theory of this rate was extremely well discussed in the studies of Magni (2010, 2011). *IRR* is defined in Equation (1) as

$$\sum_{t=0}^{n} x_t \times (1 + IRR)^{-t} = 0, \tag{1}$$

where x_t represents the cash flow at instant t; n represents the number of cash flows in the investment or financing alternative; and *IRR* represents the internal rate of return.

The *IRR* decision criterion is to accept a project when the *IRR* is greater than or equal to the cost of capital. Researchers have stated that *IRR* is not a good decision tool because it does not help a decision maker choose the best investment (Zhang 2005). Due to potential flaws of the *IRR*, academics have a preference for the net present value (*NPV*) method (Osborne 2010). However, practitioners utilize *IRR* more frequently than *NPV* to analyze their investments (Magni 2014). Furthermore, there are researchers who assert that the *NPV* versus the *IRR* debate only exists because of a misreading of *IRR*'s real proposition (Johnstone 2008; Tang and Tang 2003).

Studies have shown that when a project has a number of nonconventional cash flows (i.e., alternating inflows and outflows), multiple *IRR*s occur (Johnstone 2008). Ben-Horin and Kroll (2012) point out that multiple *IRR*s are difficult to find computationally and the accept–reject rule for deciding whether to implement a project implies a confusing decision criterion. With multiple *IRR*s, there will be more than one value to analyze, turning the *IRR* into a problem instead of a solution. Magni (2013) highlights old and new problems, including the following ones:

- There is no *IRR* when the project has its first cash flow with a sign that is opposite to its last cash flow (Magni 2013).
- When the cost of an investment's capital varies over time, *IRR* cannot be calculated because of the impossibility of comparing the *IRR* with the sequence of capital costs (Magni 2013).
- The *IRR* method does not recognize an investment's magnitude when comparing mutually exclusive projects (Magni 2013).
- The *IRR* does not measure the return on initial investment; rather, it only measures the return regarding the total (*IRR* implied) capital (Magni 2013).
- The *IRR* method ignores the project's operational lifespan (Magni 2013).
- The *IRR* does not consider accounting information (Magni 2013).
- Makeham's formula, which apparently solves the multiple *IRR* problem, actually fails if the loan has varying interest rates (Magni 2013).

Although frequently used to analyze investment decisions, *IRR* can also be used to calculate the financing cost of a loan. For this purpose, it is assumed that the value of the loan is x_0 and that x_t, $t = 1, 2, \ldots, T$ is the loan's installment. In light of the considerations above, the aim of this article is to discuss the prepurchase financing pool as a type of financing and to analyze the uses and pitfalls of the *IRR*.

The prepurchase financing pool

The history of the prepurchase financing pool in Brazil goes back to 1960, when a number of employees of the BCB decided to create a group of their friends to accrue enough money so that each participant would be able to sequentially buy a car. This was introduced to overcome the lack of credit supply at that time (ABAC 2009). A prepurchase financing pool may be understood as a community that shares money or a group of people or a collective of pooled companies making contributions to create a financing account. Each participant can then use the savings to buy goods or property (Kerr 2011). This type of loan is quite common in Brazil and its importance can be seen more clearly in Figure 1, which shows an increase in the number of prepurchase financing pools in recent years.

Prepurchase financing pools are mostly organized by private banks. These institutions follow the rules and guidelines of the BCB, which has established six different categories of underlying assets for prepurchase financing pools. As Kerr (2011) pointed out, these include

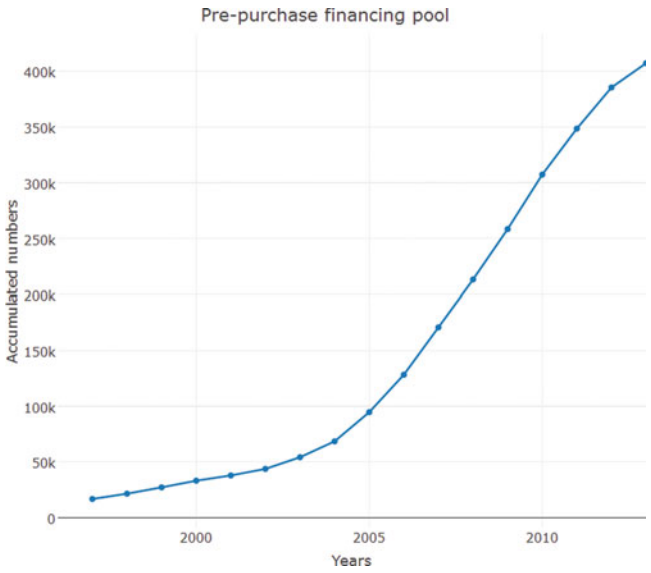

Figure 1. Accumulated number of prepurchase financing pool in Brazil. Adapted from data available on the BCB's website (see http://www.bcb.gov.br/?consorciobd).

(i) real estate, (ii) large transportation vehicles, (iii) cars, (iv) motorcycles, (v) air tickets, and (vi) other. The prepurchase financing pool, as a type of financing, is widely used as an alternative to loans. It helps people who participate in the pools to get the money they need to buy a car, a house, or anything else. This financing method is usually cheaper than a loan in terms of nominal interest rates. However, the people participating in the pool do not get their money immediately. Every month, they pay an installment, and every month a lottery takes place at which time one of the participants gets all of the money from the pool, which should be enough to buy the underlying object of the pool. The participants also get the money if an auction is held; the one who submits the highest bid gets the money for that period. The way the participants get the money is established in the contract. This process repeats until every participant is able to buy the asset (ABAC 2014). Kerr (2011) pointed out that the number of participants in the prepurchase financing pool depends on the number of quotes each participant has to pay, which is also the number of periods that the transaction will last.

As the years have passed, the prepurchase financing pool has become more popular in Brazil. It is important to highlight that financing pools are in demand due to the scarcity of available credit and the high interest rates associated with traditional loans. For instance, at the end of 2014 the government bond yield was 11.75% per annum and overdraft interest rates could reach 12% a month (BCB 2014d). The fact that a participant in a pool does not immediately get the money has never been a problem for those using this financing method; some participants consider this financial product an investment opportunity. The periodic installments can be considered forced savings that, in the end, can be used to buy a desired asset. The financial institutions allowed to sell and arrange this financing are the prepurchase financing pool managers. These institutions are required to report to the BCB and are accountable for their actions (BCB 2014c). In August 2014, there were 190 registered prepurchase financing pool managers in the BCB database (BCB 2014b).

Because prepurchase financing pools are a widespread financing method, it is important to analyze whether popular tools for investment decisions are useful. The tool that we will study in this article is the *IRR*.

IRR is a decision-making tool that helps investors calculate the real return rate of the project under analysis (Karmperis et al. 2012). In this study, we will explore the reasons why the *IRR* is not useful when assessing prepurchase financing pools. More particularly, prepurchase financing pools have nonconventional cash flows and, therefore, this type of financing presents (i) multiple rates, (ii) no rates, and (iii) ambiguous financial meaning across the PPFP's loans.

The calculation of the prepurchase financing pool rate of return depends on the value required to buy the asset, the management rate charged by the prepurchase financing pool manager, and the fund destined to protect the group's money from any situation that could negatively affect the pool. The installments are calculated as shown in Equation (2):

$$P_t = AV_t \times \frac{100\%}{n} + AV_t \times \frac{TMR}{n} + AV_t \times \frac{TRF}{n}, \tag{2}$$

where P_t represents the installment in period t; AV_t represents the asset value in period t; n represents the total number of periods; TMR represents the total management rate; and TRF represents the total reserve fund.

In the next section, we will show a numerical example of a prepurchase financing pool and its relation with the *IRR* and the *AIRR*.

The prepurchase financing pool, the *IRR* and the *AIRR*

To simulate the position of a participant in a prepurchase financing pool, we considered the following situation:

- The value of the asset being financed is $200,000.00,
- The management rate in this pool is 8.00%,
- The reserve fund is 2.00%,
- There are 10 participants.

Based on Equation (2), the value of the installment will be $22,000.00 ($200,000.00 \times (\frac{100\%}{10} + \frac{0.08}{10} + \frac{0.02}{10})$) or each participant will pay 10 installments of $22,000.00 to be able to finance an asset costing $200,000.00. Using these data, Table 1 presents the net cash flows for all participants in the prepurchase financing pool; that is, the people who will buy the underlying asset in the all periods of the fund.

Taking into account data from Table 1 and Figure 2, we show that the *IRR* can be ambiguous. Depending on the participant, the *IRR* can represent a return rate or a borrowing rate (Hazen 2003). In other situations, the *IRR* cannot be calculated or can lead to multiple values.

Most recently, the use of *AIRR* as an alternative to the *IRR* has become popular. The *AIRR* can be calculated as shown in Equation (3) (Magni 2010):

$$AIRR = r + \frac{NPV\,(1+r)}{-D}, \tag{3}$$

where NPV represents the net present value or $\sum_{t=0}^{T} x_t (1+r)^{-t}$; r is the cost of capital or the maximum attractive financing rate; and D represents the financed amount distributed to the participants of the PPFP. It is important to emphasize that a positive denominator (D) means that the project is an investment. On the other hand, a negative denominator means that the project is a financing vehicle.

Magni (2010) states that the *AIRR* correctly points to the best project and helps the decision maker decide between two or more mutually exclusive projects. As defined by Magni (2010), the *AIRR* shows that for any investment stream of cash flows, the relationship between the

Table 1. *IRR* and *AIRR* for all participants.

	Net cash flow of participants($)									
Period	1	2	3	4	5	6	7	8	9	10
0	—	—	—	—	—	—	—	—	—	—
1	178,000.00[a]	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)
2	(22,000.00)	178,000.00[a]	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)
3	(22,000.00)	(22,000.00)	178,000.00[a]	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)
4	(22,000.00)	(22,000.00)	(22,000.00)	178,000.00[a]	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)
5	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	178,000.00[a]	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)
6	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	178,000.00[a]	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)
7	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	178,000.00[a]	(22,000.00)	(22,000.00)	(22,000.00)
8	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	178,000.00[a]	(22,000.00)	(22,000.00)
9	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	178,000.00[a]	(22,000.00)
10	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	(22,000.00)	178,000.00[a]
D	(178,000.00)	(178,000.00)	(178,000.00)	(178,000.00)	(178,000.00)	(178,000.00)	(178,000.00)	(178,000.00)	(178,000.00)	(178,000.00)
NPV (20%)	74,432.28	46,654.50	23,506.35	4,216.23	(11,858.87)	(25,254.79)	(36,418.06)	(45,720.78)	(53,473.05)	(59,933.27)
First *IRR*	2.18%	2.86%	4.17%	8.53%	—	—	−7.86%	−4.01%	−2.78%	−2.14%
Second *IRR*	—	694.70%	124.61%	36.57%	—	—	− 26.78%	− 55.48%	− 87.42%	—
AIRR(20%)[b]	− 30.18%	− 11.45%	4.15%	17.16%	27.99%	37.03%	44.55%	50.82%	56.05%	60.40%

[a] This corresponds to the difference between 200,000.00 and 22,000.00.

[b] $AIRR_{\text{Participants 1}} = 0.20 + \frac{74,432.28 \times (1+0.20)}{-178,000.00}$.

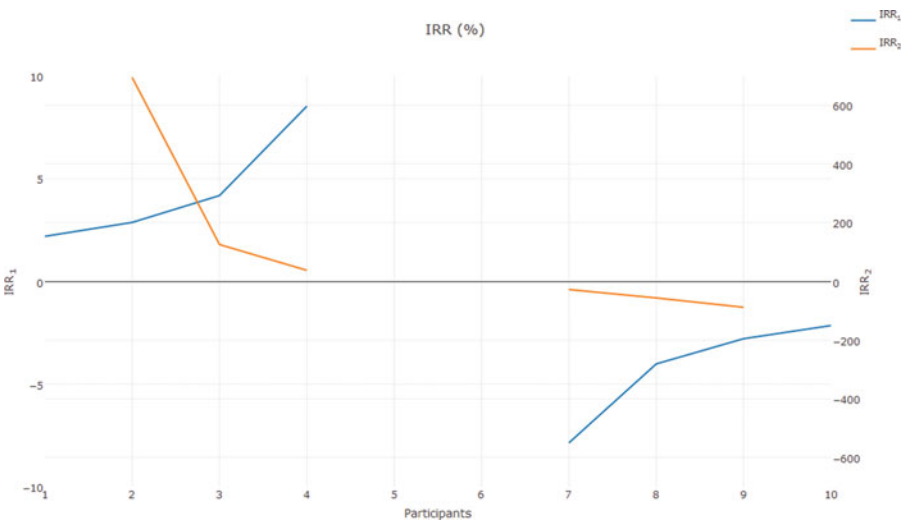

Figure 2. *IRR*s of the numerical example.

average return, the established return rate, and investment outflows always reflects a real rate of return.

The *AIRR* always follows the *NPV*, agreeing with its decision criterion. Other relevant characteristics of the *AIRR* are that it is computationally simple, it solves the problem of complex valued roots of polynomials, and it allows a direct economic interpretation.

In this article, we establish the financing flow as the distributed installments along the financing period. This means that the total amount received by the participant in the prepurchase financing pool is the $-D$ in Equation (3). After taking this into account, the *IRR* and the *AIRR* are shown in Table 1.

Considering Table 1, there are

- Two cases of unique *IRR*s: Participant 1 (borrowing rate), participant 10 (return rate);
- Six cases of multiple *IRR*s: Participants 2, 3, 4, 7, 8, and 9; and
- No *IRR* in two cases: Participants 5 and 6.

It may be shown that that participants 2, 3, and 4 are borrowers according to the smaller IRR (negative overall capital) and are lenders according to the greater *IRR* (positive overall capital; see Hazen [2003] for the definition of net investment and net financing). The use of an *AIRR* in this case indicates that the best option is first (Participant 1) and that the cost of other options is related to the fact that each participant receives the total amount to buy the underlying object at different dates. Results imply that an *AIRR* allows the comparison of different participants (e.g., Participant 1 and Participant 10). An *AIRR* also identifies the higher cost of operations as the date of distribution increases, which in turn is consistent with the results using the *NPV*.

Real-life prepurchase financing pool situations

In this section, two real-life prepurchase pool situations are shown. In the first situation, the prepurchase financing pool is related to an apartment that costs R$ 250,000.00 (Brazilian currency). The total management fee is 17.5%, and the reserve fund is 5.16%. The total number of installments is 200 and each installment is R$ 1,533.26. Taking these values into account, in Table 2, we detail the first payment from each participant.

Table 2. First real example.

	Net cash flow of participants($)									
Period	1	2	3	4	5	6	7	8	⋯	200
0										
1	248,466.74	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	⋯	(1,533.26)
2	(1,533.26)	248,466.74	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	⋯	(1,533.26)
3	(1,533.26)	(1,533.26)	248,466.74	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	⋯	(1,533.26)
4	(1,533.26)	(1,533.26)	(1,533.26)	248,466.74	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	⋯	(1,533.26)
5	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	248,466.74	(1,533.26)	(1,533.26)	(1,533.26)	⋯	(1,533.26)
6	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	248,466.74	(1,533.26)	(1,533.26)	⋯	(1,533.26)
7	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	248,466.74	(1,533.26)	⋯	(1,533.26)
8	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	248,466.74	⋯	(1,533.26)
9	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	⋯	(1,533.26)
10	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	⋯	(1,533.26)
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋱	⋮
200	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	(1,533.26)	⋯	248,466.74
IRR[a]	**0.21%**	**0.22%**	**0.22%**	**0.22%**	**0.22%**	**0.23%**	**0.23%**	**0.23%**	**⋯**	**− 0.21%**

[a] All *IRRs* are presented in Table 4.

In Table 3, we show only the first 8 periods and the first eight participants chosen in the lottery. A second example takes into account a car that costs R$ 100,000.00. This amount is received by each participant when chosen in the lottery. This PPFP has a management fee of 20% and a reserve fund of 2.5%. The total number of installments is 180 and the value of each one is R$ 680.56. Table 3 shows the first payments of each participant.

The first situation was simulated based on a product from a private prepurchase financing pool company. In this article, we cannot publish the name of the company to avoid violating any copyrights. We simulated using data from a public bank that offers the prepurchase financing pool for vehicles in the second situation. In a similar way, these two situations (Tables 2 and 3) show the same problem identified earlier in Table 1. In other words, as shown in Tables 4 and 5, we calculate the *IRR* for all participants in both situations and the following cases were identified: (i) some participants have one *IRR*, (ii) some participants have multiple *IRR*s, and (iii) some participants do not have *IRR*s. It would also be emphasized that only

Table 3. Second real example.

	Net cash flow of participants									
Period	1	2	3	4	5	6	7	8	⋯	180
0										
1	99,319.44	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	⋯	(680.56)
2	(680.56)	99,319.44	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	⋯	(680.56)
3	(680.56)	(680.56)	99,319.44	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	⋯	(680.56)
4	(680.56)	(680.56)	(680.56)	99,319.44	(680.56)	(680.56)	(680.56)	(680.56)	⋯	(680.56)
5	(680.56)	(680.56)	(680.56)	(680.56)	99,319.44	(680.56)	(680.56)	(680.56)	⋯	(680.56)
6	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	99,319.44	(680.56)	(680.56)	⋯	(680.56)
7	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	99,319.44	(680.56)	⋯	(680.56)
8	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	99,319.44	⋯	(680.56)
9	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	⋯	(680.56)
10	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	⋯	(680.56)
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋱	⋮
180	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	(680.56)	⋯	99,319.44
IRR[a]	**0.24%**	**0.24%**	**0.24%**	**0.24%**	**0.25%**	**0.25%**	**0.25%**	**0.26%**	**⋯**	**−0.23%**

[a] All *IRRs* are presented in Table 5.

Table 4. All *IRRs* (%) from first real example.

Participants	IRR_1	IRR_2	Participants	IRR_1	IRR_2	Participants	IRR_1	IRR_2	Participants	IRR_1	IRR_2	Participants	IRR_1	IRR_2
1	0.2131	—	**41**	0.3852	5.4927	**81**	—	—	**121**	—	—	**161**	− 5.4452	− 0.3758
2	0.2154	16,104.5067	**42**	0.3937	5.2408	**82**	—	—	**122**	—	—	**162**	− 5.6964	− 0.3681
3	0.2178	1,123.6258	**43**	0.4026	5.0020	**83**	—	—	**123**	—	—	**163**	− 5.9614	− 0.3608
4	0.2202	407.8003	**44**	0.4119	4.7750	**84**	—	—	**124**	—	—	**164**	− 6.2415	− 0.3539
5	0.2228	226.0727	**45**	0.4218	4.5589	**85**	—	—	**125**	—	—	**165**	− 6.5380	− 0.3472
6	0.2253	150.1030	**46**	0.4322	4.3527	**86**	—	—	**126**	—	—	**166**	− 6.8524	− 0.3408
7	0.2279	109.8274	**47**	0.4433	4.1556	**87**	—	—	**127**	—	—	**167**	− 7.1866	− 0.3346
8	0.2306	85.3198	**48**	0.4551	3.9668	**88**	—	—	**128**	—	—	**168**	− 7.5424	− 0.3287
9	0.2334	69.0115	**49**	0.4676	3.7856	**89**	—	—	**129**	—	—	**169**	− 7.9219	− 0.3230
10	0.2362	57.4594	**50**	0.4810	3.6112	**90**	—	—	**130**	—	—	**170**	− 8.3276	− 0.3176
11	0.2391	48.8910	**51**	0.4954	3.4430	**91**	—	—	**131**	—	—	**171**	− 8.7623	− 0.3123
12	0.2421	42.3078	**52**	0.5109	3.2804	**92**	—	—	**132**	—	—	**172**	− 9.2290	− 0.3072
13	0.2451	37.1072	**53**	0.5277	3.1228	**93**	—	—	**133**	—	—	**173**	− 9.7313	− 0.3023
14	0.2483	32.9052	**54**	0.5460	2.9694	**94**	—	—	**134**	—	—	**174**	− 10.2733	− 0.2975
15	0.2515	29.4466	**55**	0.5660	2.8197	**95**	—	—	**135**	—	—	**175**	− 10.8596	− 0.2929
16	0.2548	26.5550	**56**	0.5880	2.6730	**96**	—	—	**136**	—	—	**176**	− 11.4958	− 0.2885
17	0.2582	24.1053	**57**	0.6126	2.5285	**97**	—	—	**137**	− 1.3720	− 1.0086	**177**	− 12.1879	− 0.2842
18	0.2617	22.0061	**58**	0.6402	2.3852	**98**	—	—	**138**	− 1.5933	− 0.8776	**178**	− 12.9435	− 0.2800
19	0.2653	20.1892	**59**	0.6719	2.2422	**99**	—	—	**139**	− 1.7614	− 0.8025	**179**	− 13.7711	− 0.2760
20	0.2690	18.6029	**60**	0.7088	2.0977	**100**	—	—	**140**	− 1.9120	− 0.7475	**180**	− 14.6809	− 0.2721
21	0.2728	17.2071	**61**	0.7532	1.9493	**101**	—	—	**141**	− 2.0546	− 0.7038	**181**	− 15.6850	− 0.2683
22	0.2768	15.9704	**62**	0.8089	1.7929	**102**	—	—	**142**	− 2.1930	− 0.6674	**182**	− 16.7978	− 0.2646
23	0.2808	14.8680	**63**	0.8854	1.6191	**103**	—	—	**143**	− 2.3297	− 0.6362	**183**	− 18.0369	− 0.2610
24	0.2850	13.8796	**64**	1.0189	1.3911	**104**	—	—	**144**	− 2.4661	− 0.6088	**184**	− 19.4233	− 0.2575
25	0.2893	12.9889	**65**	—	—	**105**	—	–	**145**	− 2.6034	− 0.5846	**185**	− 20.9830	− 0.2541
26	0.2938	12.1826	**66**	—	—	**106**	—	—	**146**	− 2.7424	− 0.5628	**186**	− 22.7481	− 0.2508
27	0.2984	11.4495	**67**	—	—	**107**	—	—	**147**	− 2.8838	− 0.5430	**187**	− 24.7584	− 0.2476
28	0.3032	10.7804	**68**	—	—	**108**	—	—	**148**	− 3.0282	− 0.5249	**188**	− 27.0644	− 0.2445
29	0.3081	10.1673	**69**	—	—	**109**	—	—	**149**	− 3.1762	− 0.5083	**189**	− 29.7298	− 0.2415
30	0.3133	9.6038	**70**	—	—	**110**	—	—	**150**	− 3.3284	− 0.4930	**190**	− 32.8368	− 0.2385
31	0.3186	9.0841	**71**	—	—	**111**	—	—	**151**	− 3.4853	− 0.4787	**191**	− 36.4916	− 0.2356
32	0.3241	8.6035	**72**	—	—	**112**	—	—	**152**	− 3.6475	− 0.4655	**192**	− 40.8324	− 0.2328
33	0.3298	8.1577	**73**	—	—	**113**	—	—	**153**	− 3.8155	− 0.4530	**193**	− 46.0392	− 0.2301
34	0.3358	7.7431	**74**	—	—	**114**	—	—	**154**	− 3.9898	− 0.4414	**194**	− 52.3418	− 0.2274
35	0.3420	7.3565	**75**	—	—	**115**	—	—	**155**	− 4.1711	− 0.4304	**195**	− 60.0165	− 0.2248
36	0.3484	6.9953	**76**	—	—	**116**	—	—	**156**	− 4.3601	− 0.4200	**196**	− 69.3320	− 0.2223
37	0.3551	6.6570	**77**	—	—	**117**	—	—	**157**	− 4.5574	− 0.4102	**197**	− 80.3072	− 0.2198
38	0.3622	6.3393	**78**	—	—	**118**	—	—	**158**	− 4.7637	− 0.4010	**198**	− 91.8276	− 0.2173
39	0.3695	6.0405	**79**	—	—	**119**	—	—	**159**	− 4.9799	− 0.3921	**199**	− 99.3829	− 0.2150
40	0.3772	5.7588	**80**	—	—	**120**	—	—	**160**	− 5.2067	− 0.3837	**200**	− 0.2126	—

Table 5. All *IRRs*(%) from second real example.

Participants	IRR_1	IRR_2	Participants	IRR_1	IRR_2	Participants	IRR_1	IRR_2	Participants	IRR_1	IRR_2
1	0.2354	—	**46**	0.5472	3.8298	**91**	—	—	**136**	− 3.8814	− 0.5268
2	0.2382	14,493.0916	**47**	0.5662	3.6291	**92**	—	—	**137**	− 4.0815	− 0.5107
3	0.2412	1,058.6909	**48**	0.5871	3.4352	**93**	—	—	**138**	− 4.2896	− 0.4958
4	0.2442	388.9375	**49**	0.6100	3.2470	**94**	—	—	**139**	− 4.5066	− 0.4818
5	0.2473	216.6489	**50**	0.6354	3.0637	**95**	—	—	**140**	− 4.7334	− 0.4687
6	0.2505	144.1510	**51**	0.6639	2.8840	**96**	—	—	**141**	− 4.9711	− 0.4565
7	0.2537	105.5659	**52**	0.6962	2.7067	**97**	—	—	**142**	− 5.2207	− 0.4449
8	0.2571	82.0273	**53**	0.7334	2.5303	**98**	—	—	**143**	− 5.4834	− 0.4340
9	0.2605	66.3364	**54**	0.7773	2.3526	**99**	—	—	**144**	− 5.7604	− 0.4237
10	0.2641	55.2077	**55**	0.8307	2.1703	**100**	—	—	**145**	− 6.0531	− 0.4139
11	0.2678	46.9459	**56**	0.8996	1.9773	**101**	—	—	**146**	− 6.3631	− 0.4047
12	0.2715	40.5938	**57**	0.9983	1.7587	**102**	—	—	**147**	− 6.6922	− 0.3958
13	0.2754	35.5731	**58**	1.2208	1.4204	**103**	—	—	**148**	− 7.0421	− 0.3874
14	0.2794	31.5149	**59**	—	—	**104**	—	—	**149**	− 7.4151	− 0.3794
15	0.2835	28.1735	**60**	—	—	**105**	—	—	**150**	− 7.8135	− 0.3718
16	0.2878	25.3793	**61**	—	—	**106**	—	—	**151**	− 8.2401	− 0.3644
17	0.2921	23.0115	**62**	—	—	**107**	—	—	**152**	− 8.6980	− 0.3574
18	0.2967	20.9822	**63**	—	—	**108**	—	—	**153**	− 9.1907	− 0.3507
19	0.3013	19.2254	**64**	—	—	**109**	—	—	**154**	− 9.7222	− 0.3442
20	0.3062	17.6914	**65**	—	—	**110**	—	—	**155**	− 10.2972	− 0.3380
21	0.3112	16.3415	**66**	—	—	**111**	—	—	**156**	− 10.9211	− 0.3320
22	0.3164	15.1452	**67**	—	—	**112**	—	—	**157**	− 11.5999	− 0.3263
23	0.3218	14.0785	**68**	—	—	**113**	—	—	**158**	− 12.3411	− 0.3207
24	0.3273	13.1221	**69**	—	—	**114**	—	—	**159**	− 13.1531	− 0.3154
25	0.3331	12.2600	**70**	—	—	**115**	—	—	**160**	− 14.0461	− 0.3102
26	0.3391	11.4793	**71**	—	—	**116**	—	—	**161**	− 15.0320	− 0.3053
27	0.3454	10.7693	**72**	—	—	**117**	—	—	**162**	− 16.1253	–0.3004
28	0.3519	10.1209	**73**	—	—	**118**	—	—	**163**	− 17.3432	− 0.2958
29	0.3587	9.5266	**74**	—	—	**119**	—	—	**164**	− 18.7068	− 0.2913
30	0.3658	8.9801	**75**	—	—	**120**	—	—	**165**	− 20.2420	− 0.2869

31	0.3731	8.4757	**76**	—	—	**121**	—	—	**166**	− 21.9808	− 0.2827
32	0.3809	8.0089	**77**	—	—	**122**	—	—	**167**	− 23.9630	− 0.2786
33	0.3889	7.5756	**78**	—	—	**123**	− 1.4005	− 1.2061	**168**	− 26.2390	− 0.2746
34	0.3974	7.1721	**79**	—	—	**124**	− 1.7283	− 0.9885	**169**	− 28.8731	− 0.2708
35	0.4063	6.7956	**80**	—	—	**125**	− 1.9390	− 0.8915	**170**	− 31.9477	− 0.2670
36	0.4157	6.4431	**81**	—	—	**126**	− 2.1242	− 0.8239	**171**	− 35.5702	− 0.2634
37	0.4255	6.1125	**82**	—	—	**127**	− 2.2985	− 0.7713	**172**	− 39.8809	− 0.2599
38	0.4359	5.8015	**83**	—	—	**128**	− 2.4678	− 0.7280	**173**	− 45.0632	− 0.2564
39	0.4469	5.5083	**84**	—	—	**129**	− 2.6354	− 0.6914	**174**	− 51.3538	− 0.2531
40	0.4586	5.2312	**85**	—	—	**130**	− 2.8031	− 0.6595	**175**	− 59.0417	− 0.2498
41	0.4709	4.9686	**86**	—	—	**131**	− 2.9726	− 0.6314	**176**	− 68.4193	− 0.2467
42	0.4841	4.7193	**87**	—	—	**132**	− 3.1449	− 0.6063	**177**	− 79.5475	− 0.2436
43	0.4982	4.4818	**88**	—	—	**133**	− 3.3211	− 0.5837	**178**	− 91.3696	− 0.2406
44	0.5133	4.2551	**89**	—	—	**134**	− 3.5020	− 0.5631	**179**	− 99.3147	− 0.2377
45	0.5296	4.0381	**90**	—	—	**135**	− 3.6885	− 0.5442	**180**	− 0.2348	—

*IRR*s greater than −100% are shown in Tables 4 and 5 and that in these real-life cases the *IRR* notion completely collapses.

Conclusions

After analyzing the examples given in the previous section and studying both *IRR* and the prepurchase financing pools, one realizes that it is not possible to calculate the effective cost of this type of financing using the *IRR* methodology. The main reason is that the pool has many participants, and each one has a different likelihood of having the money at a given period t. This means that the prepurchase financing pool method presents various sign changes in cash flows that imply multiple *IRR*s or even the impossibility of calculating an *IRR* because of the large number of periods and participants.

Therefore, because this financing method is extremely popular in Brazil, in future studies, we intend to suggest that the *AIRR* approach should be deeply analyzed as a potential solution for measuring the effective cost of prepurchase financing pools.

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