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Short Communication

A graph-theoretic perspective on the links-to-concepts ratio expected in cognitive maps

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ABSTRACT

Strategic options development and analysis (SODA) has maintained that it expects a links-to-concepts ratio of 1.15–1.20 in cognitive maps. This expectation is investigated from a graph-theoretic perspective in order to highlight two issues that the SODA literature has not mentioned. First, the ratio is impossible to achieve in tree-structured maps. Second, adherence to the expectation can result in minimally connected maps. Both issues are discussed with examples and calculations, and a conclusion is drawn that graph theory is a potentially rich, yet relatively untapped, source of insights for not only SODA, but for soft OR and systems thinking in general.

In a paper published relatively recently in this journal, Eden (2004) provided an overview of the possible ways in which the cognitive maps of strategic options development and analysis (SODA) – henceforth SODA maps – may be analyzed. The content of the paper is very similar to the one published by Eden et al. (1992) 12 years previously. Both papers note, early on, that the essential structure of a SODA map is that of a directed graph – or digraph – and they cite the classic texts on this subject by Harary (1972) and Harary et al. (1965). Readers wanting a concrete mathematical explanation as to why the essential structure of a SODA map is that of a digraph need only examine the axiom system of the latter as given by Harary et al. (1965, p. 9). A more recent comprehensive treatment of directed graphs – or digraphs – is provided by Bang-Jensen and Gutin (2002).

The structural similarity that SODA maps share with directed graphs has led to an increasing interest in the possible insights offered by the latter for the purposes of analyzing the former. Shaw et al. (2003) discuss domain analysis (node degree, in graph-theoretical terms), explosions (outdegree) and implosions (indegree). Langfield-Smith and Wirth (1992) draw on graph-theoretic adjacency matrices in order to compute degrees of difference between cognitive maps. Wang (1996) takes this one step further by introducing a neural network approach that can complement the use of adjacency matrices when analyzing cognitive maps. Recently, Montibeller and Belton (2006) have neatly summarized the state of the art in cognitive map analysis, with graph-theoretic approaches featuring as useful formal techniques.

There is evidence, therefore, that the abstract, and yet practically useful, field of graph theory – especially digraph theory – has something of value to contribute to the analysis of SODA maps. At the same time, it is obvious from the SODA mapping literature that its research into graph theory's analytical tools has barely begun to scratch the surface of this latter field. In this respect, SODA analysts might be interested in the approaches undertaken by social network analysis, which rely heavily on some advanced graph theory (Wasserman and Faust, 1994).

SODA analysts have also missed some basic graph-theoretic insights even within the relatively elementary interdisciplinary research undertaken thus far. Consider, for instance, Eden's (2004) claim that, for SODA maps, the typical expected ratio of links-to-concepts lies within a fairly constant interval of 1.15–1.20 – a claim echoed at least since 1992 in the SODA literature (Eden et al., 1992; Eden and Ackermann, 1998; Bryson et al., 2004; Ackermann and Eden, 2005). In all cases, the claim is said to be based upon experience (empirical evidence) of numerous mappings across different applications. Graph theory raises two issues regarding the ratio.

First, the ratio is impossible to achieve in SODA maps that contain no feedback loops and in which there exists exactly one path between any two concepts. Structurally, such maps are what graph theory calls 'trees' (Aldous and Wilson, 2000, pp. 138–181). Indeed, as the number of concepts increases, the links-to-concepts ratio increases asymptotically without ever reaching the value of 1. This is due to the theorem that if a graph G is a tree of order p and size q,

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then $q = p-1$ (where the order of a graph is the number of vertices, and its size is the number of edges) (Chartrand, 1977, p. 11, 82). It follows that $q/p < 1$, that is, the links-to-concepts ratio will always be less than 1.

The SODA literature makes no mention of the fact that the claimed range of the links-to-concepts ratio is unreachable in tree structures. Yet, maps in the form of trees are not uncommon. Eden and Simpson (1989), for instance, provide one as part of their demonstration case study in the first edition of *Rational Analysis for a Problematic World* (Rosenhead, 1989) – see the first map of their Fig. 3. Eden (2004) (also see Eden et al. (1992)) discusses discarding links that, whilst acting as summaries of more detailed paths, do not offer a different causality to the indirect linkages. An analysis that takes this to the limit will yield a tree-structured map. Tree structures are also useful as subgraphs (submaps) that can assist in the analysis of wider structures (see, for example, Fig. 2 of Eden et al. (1992); for an introduction to the usefulness of subgraphs in graph theory, see Berge (1962, pp. 152–164)). They are also evident in hierarchic strategy maps (see Fig. 3 in Bougon (1992)). Furthermore, it is not unusual for tree structures to be the basis of the first few iterations of map design. Fig. 1(a), for example, shows the result of initial attempts to map a situation described by Rodrigo Vilaça, the director of the National Association of Railway Transporters of Brazil (Vilaça, 2005). Only when additional contextual information was obtained did the feedback loop in Fig. 1b appear as a logical inclusion.

The second issue raised by graph theory concerning the ratio is that, as the number of concepts increases in a map, maintaining a links-to-concepts ratio of 1.15–1.20 leads to an exponentially lesser overall map density, where density is measured by the ratio of links to maximum possible links. It is the case that by the time one reaches only 13 concepts in a SODA map, whilst choosing to maintain a links-to-concepts ratio in the stipulated range of 1.15–1.20, the overall map density is less than 10% (out of interest, it is worth noting that for undirected graphs, the number of nodes is 25). Furthermore, by the time one reaches 100 concepts or more, whilst still choosing to maintain the stipulated links-to-concepts ratio, the overall density is less than 1%; that is, less than 1% of all possible connections between concepts have been made.

This may not necessarily be troublesome – it all depends on the context of the analysis. The danger lies in analysis that reduces a map to only 1% of its connective possibilities for the sake of maintaining what can easily be interpreted as a guideline or a benchmark links-to-concepts ratio. The reason why the ratio might be so interpreted, and why attempts might be made to artificially maintain it, lies in it having been stipulated as 'expected' by the SODA literature – see, for example, Eden and Ackermann (1998), as well as Eden (2004).

There is some evidence that the stipulated expectancy of the ratio has indeed been understood as a benchmark, and that, due to such understanding, the ratio causes some discomfort to SODA users, especially to those outside of the mainstream literature who look to it for clear and tested practical orientation. Consider, for example, Carbonara and Scozzi (2006) who describe an analysis based upon four maps, one of which happens to be a tree. In analyzing their overall results, they note that their maps:

> have an average number of 16 concepts (ranging from eight to 20) and average link/concept ratios of about one (ranging from 0.8 [the ratio associated with the tree map] to 1.3). The number of concepts is quite low, but the link/concept ratio is slightly superior to the values *proposed* in the literature (Eden et al., 1992). [authors' reference, italics added]

Given the low number of concepts as well as a links-to-concepts ratio that overshoots what the literature *proposes*, the authors believe that, in their case, complexity may be evaluated by simultaneously considering the ratio *along with* the number of concepts. Unfortunately, the manner in which this synthesis may yield meaningful analytical results is not further explained by the authors. One is left wondering: is their having reached a *cul-de-sac* a failure that reflects their own abilities? Or is it a reflection of the SODA literature in having failed to clarify the structural underpinnings on which a very basic analytical tool rests?

In the wider scheme of things, the relevance of graph theory does not stop with SODA. Systems thinking, for instance, of which SODA and SSM are exemplars, has yet to tap into the analytic – and visual (Di Battista et al., 1999) – potential of graph theory and its related areas, despite the fact that it deals extensively with

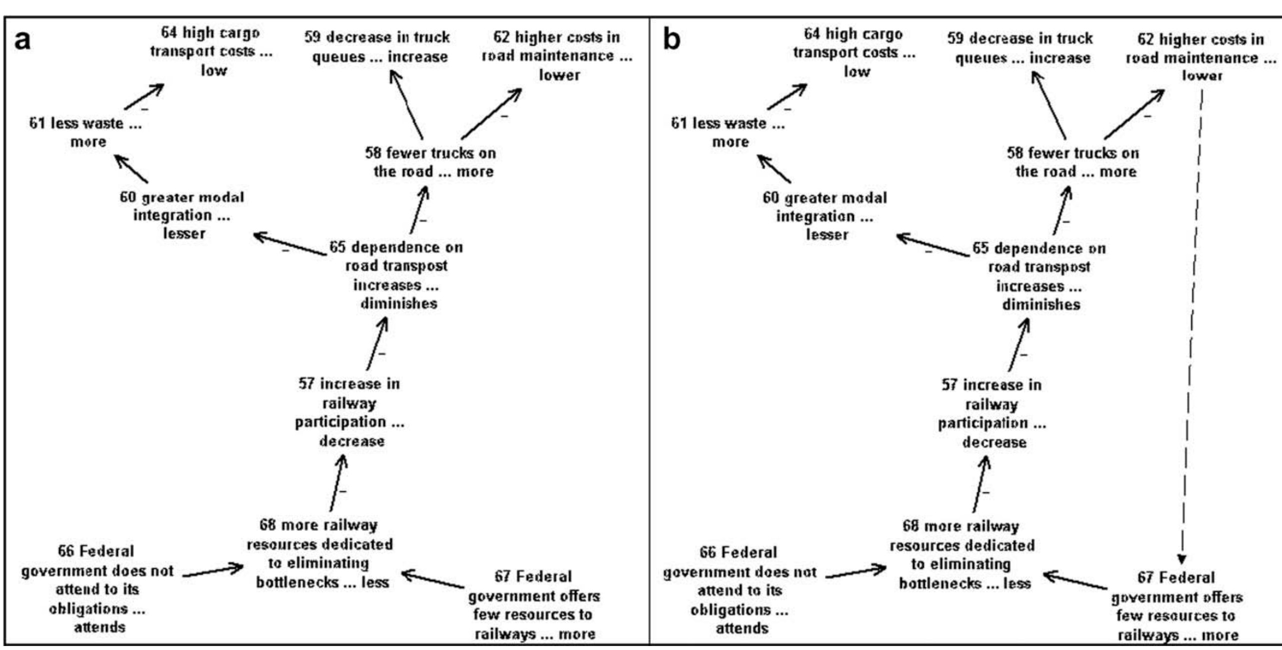

Fig. 1. Development of a cognitive map. (a) Initial tree-structured map, (b) identification of a feedback loop.

systemic digraph structures. Graph-theoretic analysis is amenable whenever a system of interrelated elements is posited – in other words, it speaks to the structural essence of systems thinking. Now, systems thinking has itself made considerable analytical headway – aside from SODA and SSM, one need only consider the power of system dynamics (Sterman, 2000) or the identification of systems archetypes (Braun, 2002). Interdisciplinary *graph-theoretic-systems thinking* research promises a rich complementary set of useful practical results. It is not a question of promoting such interdisciplinarity simply due to the fact that both fields address elements in interrelation. The need arises particularly when methodological questions in systemic thinking, such as those raised in the above examples, point to the practical advantages of considering graph theory.

In the Preface to their introductory book on graph theory, Buckley and Lewinter (2003) note that '[w]ith the advent of operations research in the 20th century, graph theory has risen several notches in the esteem in which it is held'. Be that as it may, it would appear that soft OR and systems thinking appear to have dismissed its relevance to their own purposes. This is unfortunate, especially given that graph theory can offer a degree of quantitative analysis that can complement the evaluation of soft OR and systems thinking models. SODA itself has made limited use of graph-theoretic insights. Earlier, a graph-theoretic perspective on a very basic 'expected' result of SODA map analysis outlined potentially detrimental practical consequences of following this guideline under certain circumstances. One can only conclude that a more thorough investigation of graph theory and its analytical capabilities in the context of soft OR would be to everybody's advantage.

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