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Efficiency in the Brazilian banking system using data envelopment analysis☆

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ABSTRACT

The objective of this paper is to evaluate bank efficiency in the period from 2012 to 2016 by applying Data Envelopment Analysis (DEA) in a dataset of 37 Brazilian banks provided by the Brazilian Central Bank. We have explored three gaps in research conducted with Brazilian banks by using the intermediation approach to select variables, by analysing the main causes of bank inefficiency and by identifying how inefficient banks in scale can improve their efficiency. Brazilian banks presented an average efficiency of 51.4% for the Charnes, Cooper and Rhodes (CCR) model and 69.8% for the Banker, Charnes and Cooper (BCC) model. The largest banks have performed well in regards to Pure Technical Efficiency (PTE), but failure to operate at the optimal scale level has impaired Technical efficiency (TE), jeopardizing the position of these banks in the efficiency ranking. These banks, in the majority, presented decreasing returns to scale, while the smaller banks had increasing returns to scale. Inefficiency of Brazilian banks is slightly more related to technical and administrative issues than to the scale of operations, although the banks have many opportunities for improvement in this second aspect, especially the larger banks. Ribeirão Preto Bank was the most efficient bank in the group, followed by Cooperativa Sicredi Bank and Alfa Bank. All three banks can be considered small banks. The results indicate that the largest banks are not necessarily the most efficient ones. The efficiency of the sector could be increased if policies were adopted to increase the participation of the smallest banks in the sector, which is currently highly concentrated in the largest ten banks. Government could encourage a dilution in market share of larger banks either through fiscal stimuli among small banks or by fostering mergers and acquisitions.

1. Introduction

Banks are key elements in a country's economy. According to Tsolas and Charles (2015, p. 3491), the banking sector plays a central role in the development of the economy; therefore, problems in this segment are the focus of various studies. Svitalkova (2014, p. 664) states that it is important for countries to have a consolidated and advanced banking system, since the better its financial environment, the more competitive a nation will be.

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Considering the relevance of financial institutions, many studies have sought to evaluate the performance of banks in different countries (Schure et al., 2004; Řepková, 2014; Lin et al., 2009; Liu, 2010; Wanke et al., 2016a; Sokic, 2015; Barros & Wanke, 2014; Kamarudin et al., 2017). Among the methods to evaluate the performance of banks, efficiency frontier techniques stand out. Berger and Humphrey (1997, p. 11), for example, examined 130 studies that investigated 21 different countries to measure bank efficiency through parametric and non-parametric methods, which evidences the importance of studies on efficiency in this sector.

Svitalkova (2014, p. 645) points out that non-parametric techniques are more adequate than parametric models to rank the efficiency of banking institutions. In this context, according to Wanke et al. (2016a), Data Envelopment Analysis (DEA) is the main non-parametric technique currently used for efficiency assessment. This empirical mechanism, developed by Charnes et al. (1978), is based on a mathematical technique of measuring the efficiency of a homogeneous group of decision-making units (*DMUs*) that use the same inputs and outputs. By transforming a programming problem with infinite solutions into a linear programming approach, DEA identifies the most efficient *DMUs* and indicates what inefficient units must do to become efficient. In other words, DEA allows best practices to be identified from an efficiency frontier (Charnes et al., 1994, pp. 7–8).

The first study to apply DEA to financial institutions was Sherman and Gold (1985), which aimed to evaluate 14 branches of a bank. These authors verified that traditional techniques for measuring performance such as profitability and transaction costs were not so appropriate because they did not take into account the complexity of the operations of each branch and did not consider the multiple outputs generated by multiple inputs. After this research, the banking sector became one of the main areas of interest for the application of DEA, as depicted in studies by Assaf et al. (2011); Kwon and Lee (2015); Holod and Lewis (2011); Gulati and Kumar (2017); Sufian (2015); Pasiouras (2008).

Among studies on efficiency, Luo (2003) highlights that an important issue in the literature related to financial institutions is the evaluation of which type of return to scale (RTS) banks' experience. Banks can present increasing return to scale (IRS), constant return to scale (CRS) or decreasing return to scale (DRS). To verify what type of RTS a bank has, it is necessary to decompose the overall efficiency indexes. We use both Charnes, Cooper and Rhodes (CCR) and Banker, Cooper and Rhodes (BCC) model, since the first model will measure Technical Efficiency (TE), also called global efficiency, while the second model identifies Pure Technical Efficiency (PTE) related only to administrative and managerial capabilities. Scale Efficiency (SE), which is linked to the operating scale level, is then calculated by the ratio of TE to PTE. By using both the CCR and the BCC models, we achieve a better understanding of the causes of inefficiencies of the banks under analysis. Is the bank inefficient because of its administrative and managerial skills or because of its operating scale level? The application of the two models allows to answer such question, as discussed by Řepková (2014) and Yilmaz and Güneş (2015).

In spite of the high popularity of DEA in studies that aim to measure bank efficiency either by only measuring efficiency indices or by bringing a more in-depth discussion of TE, PTE, SE and RTS types of efficiency, Wanke and Barros (2014) argue that the vast majority of studies focus on the United States and the European Union. Among the research carried out in Brazil, we can cite Périco et al. (2008); Ceretta and Niederauer (2001); Souza and Macedo (2009) and Wanke and Barros (2014).

Périco et al. (2008) sought to verify whether the largest banks were the most efficient by applying the BCC model. Ceretta and Niederauer (2001) evaluated the profitability and efficiency of 144 financial conglomerates using a two-stage model. Souza and Macedo (2009) applied DEA to measure the performance of the 100 largest banks in activity in Brazil, from 2001 to 2005, using a composite boundary model. Wanke and Barros (2014) also measured the efficiency of Brazilian banks using a two-stage DEA model.

As Wanke and Barros (2014) have pointed out, the amount of research in the context of the Brazilian banking sector is very limited. To the best of our knowledge, none of the studies with Brazilian banks have decomposed efficiency, measuring PTE, SE, and TE, which in turn would allow a deeper understanding of the efficiency of the sector, and also have not identified what kind of return to scale banks would be presenting. These two aspects are of great relevance because, when addressed, it would be possible to suggest managers' procedures that should be taken to make the bank more efficient, i.e., indicating the cause of inefficiency (administrative issues or operating scale level). Additionally, by identifying the bank's return to scale, it would be possible to deepen the discussion about potential scale inefficiencies of the banks. By verifying whether the bank presents increasing, constant or decreasing returns, scale inefficiency can be reversed by adequately changing the scale of operations.

Finally, analysing the research in the Brazilian context, as discussed in depth in Section 2, only two studies (Staub et al., 2010; Wanke and Barros, 2014) followed the intermediation approach in the selection of the variables. This procedure, proposed by Sealey and Lindley (1977), has received great attention by researchers and is currently the predominant approach in bank efficiency studies (Fethi and Pasiouras, 2010, p. 191)[1]. This perspective is based on the bank's primary function of collecting funds and converting them into loans and other profitable assets, using physical capital and labour. In this approach, the bank is seen as a financial intermediary between agents with surplus and agents with deficit of financial resources. Although the intermediation approach is widely used in the literature, few studies have applied that approach to select variables in Latin America, as evidenced in Table 1.

This research, therefore, addresses three gaps in the literature on bank efficiency not only in the Brazilian context but also in Latin America and also provides policy implications for Brazilian banks by following the intermediation approach for the selection of variables, by measuring TE, SE and PTE and, finally, by verifying the RTS of the banks.

In this context, the aim of this study is to verify which Brazilian banks were most efficient in their role as financial intermediary institutions. By analysing from 2012 to 2016, we study whether the largest banks in terms of total assets were the most efficient and what the main causes of inefficiency of the banks were. We use the database entitled “The Fifty Largest Banks”, made available by the

[1] Fethi and Pasiouras (2010) reviewed 196 studies that applied operations research techniques in the banking sector. In their review, 151 studies used the DEA, most of them following the intermediation approach.

Table 1
DEA and banking efficiency.

Study	Objective	Country	Model	Input	Output	Results
Sherman and Gold (1985).	To analyse 14 bank branches through the application of DEA.	USA.	Not specified.	Employees by branch, cost of space and expenses.	Number of transactions in each branch.	Although DEA has certain limitations, it provides important information for performance evaluation not available by other techniques.
Seiford and Zhu (1999)[a].	To evaluate the 55 largest US banks and identify the most efficient in terms of profitability and marketability[b]	USA.	Two stages.	First stage: number of employees, total assets and shareholders' equity. Second stage: profits and revenue.	First stage: profits and revenue. Second stage: market value, total return to shareholders and earnings per share.	Approximately 90% of banks were inefficient in terms of profitability and market value. The size of the bank has decreasing returns of scale regarding market value and positive returns of scale related to profitability.
Ceretta and Niederauer (2001).	To investigate the competitive position of 144 financial conglomerates in the banking sector through the matrix representing profitability versus productive efficiency.	Brazil.	Two stages.	Not specified.	Not specified.	Larger conglomerates have far superior operational efficiency and profitability to other banks.
Luo (2003).	To analyse TE, PTE and SE regarding the profitability and marketability of 245 US banks in addition to verifying RTS and whether the geographical area of the bank has an impact on efficiency.	USA.	Two stages, CCR–1 and BCC–1.	First stage: number of employees, total assets and shareholders' equity. Second stage: profits and revenue.	First stage: profits and revenue. Second stage: market value, total return to shareholders and earnings per share.	The largest source of banks inefficiency is in marketability; larger banks more frequently presented DRS, while the smaller ones presented mostly IRS.
Becker et al. (2003).	To analyse the relative efficiency of Brazilian banks, taking into account investments made in IT.	Brazil.	BCC - 1.	IT investments, personnel expenses, expenses with physical structure and administrative expenses.	Net revenues from financial intermediation, service provision and international operations.	Banks that invested more in IT were more efficient globally. Foreign banks were the most efficient in the sample.
Havrylchyk (2006)[a].	To investigate the efficiency of banks in Poland from 1997 to 2001.	Poland.	Not specified.	Number of employees, deposits and fixed assets.	Loans, government bonds and off-balance-sheet elements[c] (Assets or debts that do not appear on the company's balance sheet. For banks, one example is operating leasing).	On average, foreign banks were more efficient than Polish banks. The efficiency indices practically did not increase during the period.
Ariff and Can (2008)[a].	To analyse the cost and profit efficiency of 28 Chinese commercial banks from 1995 to 2004.	China.	CCR and Tobit.	Total deposits, number of employees and fixed assets.	Investments and total loans.	The level of profit efficiency was lower than the level of cost efficiency. Medium-sized banks were the most efficient.
Périco et al. (2008).	To verify whether the largest banks are also the most efficient in terms of the use of their resources.	Brazil.	BCC-2.	Shareholders' equity, total assets and total deposits.	Net income.	The largest banks are not necessarily the most efficient.
Drake et al. (2009)[a].	To examine the efficiency of the Japanese banking industry using the intermediation and production approaches as well as the profit-and-return approach.	Japan.	SBM (Slack-based measure.	Total deposits, total operating expenses, total provisions, non-interest expenses and other operating expenses	Total loans, other profitable assets, net commissions, other operating revenues and net interest income.	The approach focusing on intermediation almost always produces efficiency indices that are superior and more stable than the indices of other approaches. The efficiency levels calculated from distinct approaches presented considerable differences.
Kumar and Gulati (2009)[a].	To evaluate the efficiency, effectiveness and performance of 27 public banks operating in India.	India.	Two stages.		First stage: investments and loans. Second stage: net interest income and non-financial income.	

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Table 1 (*continued*)

Study	Objective	Country	Model	Input	Output	Results
				First stage: fixed assets, number of employees and loanable resources. Second stage: investments and loans.		Only 15% of the banks were fully efficient. The mean in the two stages was 0.9122. There was a strong correlation between effectiveness and performance. High efficiency does not mean high effectiveness. Large banks have lower performance than small ones.
Lin et al. (2009).	To evaluate 117 branches of a Taiwanese bank in 2006.	Taiwan.	SBM, CCR and BCC.	Number of employees, interest expenses, total deposits and checking deposits.	Loans, interest income, operating income and earnings.	There were more inefficient branches than efficient ones. The size of the organization did not influence the efficiency index.
Souza and Macedo (2009).	To analyse and evaluate the performance of the 100 largest banks in Brazil from 2001 to 2005, divided into four segments.	Brazil.	CCR-1 and 2 with inverted border.	Property, plant and equipment, operating costs and leverage.	Current liquidity and operational profitability.	2004 may be considered the best year for banks, while 2002 was the worst year. Banks that managed to reduce their fixed assets during the period were more efficient. In general, the largest banks in terms of assets were not the most efficient.
Liu (2010)[a].	To analyse 25 banks in Taiwan and classify them into four distinct categories, according to their competitiveness and technical improvement.	Taiwan.	Malmquist Index.	Number of employees, fixed assets and funds such as savings and deposits.	Checking accounts, short-term loans and long-term loans.	Fifteen Taiwanese banks have been increasing their technical efficiency over time, while 10 banks have showed declining technical efficiency.
Staub et al. (2010)[a].	To estimate the cost, allocative and technical efficiencies of Brazilian banks in the post-privatization period (2000–2007).	Brazil.	Three panel models[d]	Expenses with the exception of personnel expenses, personnel expenses and interest expenses.	Total loans, investments and deposits.	Brazilian banks have a higher degree of inefficiency compared to banks in other countries. State-owned banks were more efficient than private banks, and foreign banks showed higher levels of cost inefficiency. Size is not an important variable that impacts efficiency.
Assaf et al. (2011)[a].	To analyse the efficiency of Saudi banks from 1999 to 2007.	Saudi Arabia.	Two stages.	Deposits, number of employees and fixed assets.	Loans, securities and interbank loans.	State-owned banks were more efficient than private banks, challenging the conflicts suggested by agency theory.
Řepková (2014)[a].	To examine the efficiency of Czech banks from 2003 to 2012.	Czech Republic.	DEA, using window analysis –SBM, CCR and BCC.	Deposits and personnel expenses.	Loans and net interest income.	The average efficiency for the CCR model was between 70% and 78%, while the average efficiency of the BCC model was between 84% and 89%.
Svitalkova (2014)[a].	To measure and compare the efficiency of banks in different countries within the European Union and identify the origins of inefficiency.	European Union.	Networking CCR and BCC with and without the undesired output.	Expenses with personnel, fixed assets and deposits.	Loans and net interest income. As undesirable output: provision for credit losses (PCL).	The countries considered efficient varied considerably according to the model. Banks in all countries should focus on increasing lending, while ensuring that PCL does not grow.
Wanke and Barros (2014)[a].		Brazil.	Two stages.		First stage: Administrative expenses and personnel expenses. Second stage: Equity and fixed assets.	

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Table 1 (*continued*)

Study	Objective	Country	Model	Input	Output	Results
	To evaluate the 40 largest Brazilian banks regarding optimized costs and productive efficiency, establishing a connection between these two variables.			First stage: Number of agencies and Number of employees. Second stage: Administrative expenses and Personnel expenses.		Comparatively, Brazilian banks tend to be more efficient in converting administrative expenses and personnel expenses into equity and fixed assets than in managing physical and human resources. Variables related to mergers and acquisitions, size and state-owned status are determinants for efficiency.
San-Jose et al. (2014).	To evaluate the general, social and economic efficiency of the banking system in Spain from 2000 to 2011.	Spain.	Two stages: DEA and regression.	Shareholders' equity, total assets and total deposits.	Profit, risk, social contribution, number of jobs and consumer credit.	Investment banks have overall efficiency ratios similar to other banks in Spain, although they are more efficient in the social aspect.
Zimková (2014)[a].	To analyse the efficiency of 16 banks in Slovakia through three DEA models in 2012.	Slovakia.	BCC, SBM and super-efficient SBM.	Fixed assets, deposits and number of employees.	Earning assets[e].	More than half of the banks were able to convert human capital and fixed assets into earning assets. The Pearson coefficient indicates a high correlation between the BCC model and the SBM.
Chan et al. (2015)[a].	To study the effects of market and institutional structure on bank efficiency in five large South-east Asian countries.	Asia.	Two stages: SBM and GMM[f].	Expenses, interest expenses and other non-interest expenses.	Interest income from loans, investments, income from off-balance-sheet activities and other non-financial revenues.	Government regulations in the sector should aim to increase market discipline, monitoring and transparency, however, banking activities themselves should have little intervention.
Kamarudin et al. (2015)[a]	To investigate TE, PTE and SE of 43 banks of the Gulf Cooperation Council (GCC) from 2007 to 2011.	GCC countries.	CCR–1 and BCC–1.	Total assets and deposits.	Loan and income.	Bank inefficiency was mainly related to administrative capabilities. The largest banks presented DRS, while the smaller ones presented IRS.
Kwon and Lee (2015).	To combine two empirical data analysis techniques to assess and predict performance improvements.	USA.	Two stages: CCR-2 and Back Propagation Neural Network (BPNN).	First stage: Employees, Shareholders' equity and Expenses. Second stage: Deposits, Loans and Investments.	First stage: Deposits, loans and investments. Second stage: Profit.	The proposed model helps the managerial process of decision making.
Yilmaz and Güneş (2015)[a].	To measure and compare the purely technical, scale and overall efficiencies of conventional deposit banks and Islamic banks from 2007 to 2013.	Turkey.	CCR-1 and BCC-1.	Total deposits and capital.	Total loans, investments and income.	The inefficiency of Islamic banks during the period was predominantly of scale, unlike conventional banks. Globally, Islamic banks were slightly more efficient than the other banks.
Wanke et al. (2016b).	To apply a new fuzzy DEA model to evaluate bank efficiency in Mozambique from 2003 to 2011.	Mozambique.	Fuzzy.	Total expenses (except personnel) and personnel expenses.	Total deposits, pre-tax income and total credit operations.	Several aspects explain the bank efficiency in Mozambique, such as labour costs, capital costs and inflow and outflow deposits. Banks should decrease the number of employees and make initiatives to leverage capital.
Kamarudin et al. (2016)[a]	To provide new empirical evidence on the impact of country governance on the revenue efficiency of Islamic and conventional banks during the period of 2007–2011.	GCC countries.	Two stage - BCC with panel regression.	Personnel expenses and deposits.	Loan and income.	

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Table 1 (*continued*)

Study	Objective	Country	Model	Input	Output	Results
						Islamic banks were less efficient than conventional banks. Macroeconomic variables have distinct impacts in revenue efficiency, whereas greater voice and accountability, government effectiveness and rule of law improve the revenue efficiency of both Islamic and conventional banks.
Stewart et al. (2016)[a].	To analyse the efficiency of the Vietnamese banking system from 1999 to 2009 by identifying the determining variables for bank efficiency.	Vietnam.	Two stages: CCR, BCC and bootstrap.	Number of employees, deposits from other banks and client deposits.	Loans from customers, other loans and securities.	The largest banks were more efficient than the medium and small banks, with small banks being the most inefficient. As far as global efficiency is concerned, private banks were more efficient than state-owned banks.
Kamarudin et al. (2017)[a]	To examine the productivity of Islamic banks by verifying specific factors of banks, industry, and macroeconomy that can influence productivity.	Southeast Asian countries.	Two stage - BCC with Malmquist and panel regressions.	Personnel expenses, deposits and fixed assets.	Loans and investments.	The productivity of Islamic banks increased during the analysed period. Capitalisation, liquidity and world financial crises have significant influence on the productivity level of Islamic banks.

[CCR–1] CCR model, input-oriented.
[CCR–2] CCR model, output-oriented.
[BCC–1] BCC model, input-oriented.
[BCC–2] BCC model, output-oriented.
[a] The paper followed the intermediation approach.
[b] factors such as market value, earnings per share and return to investors make up marketability.
[c] Assets or debts that do not appear on the company's balance sheet. For banks, one example is operating leasing.
[d] DEA self- regressive in the error structure and Tobit.
[e] Loans and commercial papers held to maturity.
[f] Generalized method of moments.

Brazilian Central Bank (BACEN). We applied both the CCR and the BCC models, which are input oriented. In addition, we examined TE, PTE, SE and RTS indices. It is worth emphasizing that,in exploring the three existing gaps discussed above, the three research questions addressed in this paper will be also answered. By decomposing the overall efficiency index, we can identify the main cause of inefficiency of the banks, and, with the analysis of the RTS, we can provide a more in-depth discussion of the managerial implications of scale and study whether or not largest banks are the most efficient. We sought to discuss the following questions:

1. Which were the most efficient banks between 2012 and 2016?
2. Are the largest banks also the most efficient in their role as financial intermediary institutions?
3. What is the main cause of inefficiency in banks in Brazil?

In order to address these issues, this paper is structured as follows: Section 2 presents a brief history of DEA, the main theorists who contributed to the development of this technique and the two main models-CCR and BCC-used in this study. Section 3 summarizes some DEA studies in banks, evidencing the existence of the gaps explored in this work. Section 4 discusses the Brazilian financial system, highlighting the characteristics of this sector over the years and how it is today. Section 5 discusses the method used in our paper, explaining how the variables were chosen, how the DEA models were used and the choice of the orientation. We also describe the database. Section 6 contains the results of this study, which indicate that the largest banks are not the most efficient. The results imply that the allocation of financial resources, i.e., funding and investment, may be inefficient regarding the expected impact for the Brazilian economy. Finally, Section 7 presents the main conclusions and suggestions for future work.

2. DEA

2.1. Brief history and context

DEA began with Edward Rhodes' doctoral dissertation under the supervision of William W. Cooper at Carnegie Mellon University. Seiford (1996, pp. 99–100) points out that DEA builds on the works from Afriat (1972); Aigner and Chu (1968); Shephard (1970); Debreu (1951); Farrel (1957); Koopmans (1952) and Pareto (1927) as well as on the algebraic manipulations of Charnes and Cooper (1962), which made it possible to transform linear programming with infinite solutions into conventional linear programming.

After the publication of the study from Charnes et al. (1978) in the *European Journal of Operational Research*, DEA became quite popular among academics and analysts. Charnes et al. (1994, p. 10) pointed out that approximately 400 articles, books and dissertations involving DEA were published between 1978 and 1994. Emrouznejad et al. (2008, p. 152), analysing a longer period of time, found 4000 published articles. If dissertations and other materials were considered, the number of related materials could have reached 7000 publications in 30 years under this model.

Several reasons have contributed to the great popularity of DEA: it allows for analysis of efficiency at the *DMU* level, comparison of each *DMU* to the group, analysis of whether the *DMU* is efficient, identification of the causes of inefficiency and how the *DMU* can improve its efficiency (Řepková, 2014, p. 589), the establishment of an efficiency frontier based on empirical data rather than on theoretical notes and the possibility of incorporating various variables in different types of measures (Svitalkova, 2014, p. 645).

2.2. CCR model

In 1978, Charnes et al. (1978, pp. 430–435) proposed the CCR model, which consists of a nonparametric mathematical linear programming technique that makes it possible to work with multiple inputs and multiple products as well as to identify best practices from an efficient frontier composed of efficient *DMUs*. The model can also indicate directions for inefficient *DMUs* to become efficient. In order to work with linear programming, the authors used the algebraic manipulations elaborated by Charnes and Cooper (1962) to transform a nonlinear programming problem into a linear programming problem.

The CCR model extended Farrel (1957)'s study, starting from multiple inputs and only one output to multiple inputs and outputs by generalizing the efficiency ratio to a single *DMU* and transforming the multiple inputs and outputs of each *DMU* into a single virtual input and output (Charnes et al., 1994, p. 4). This model is based on constant returns of scale and is therefore also known as CRS model. The basic equations of the input and output-oriented CCR model are presented in Eqs. (1) and (2), respectively.

$$\max h_k = \sum_{r=1}^{m} u_r\, y_{rk} \tag{1}$$

Subject to:

$$\begin{aligned} &\sum_{i=1}^{n} v_i\, x_{ik} = 1 \\ &\sum_{r=1}^{m} u_r\, y_{rj} - \sum_{i=1}^{n} v_i\, x_{ij} \le 0, \forall\ j \\ &u_r, v_i \ge 0, \forall\ r, i \end{aligned}$$

with:

$y = outputs$; $x = inputs$
$u, v =$ weights;
$r = 1, \ldots, m$; $i = 1, \ldots, n$;
$j = 1, \ldots, N$

CCR Model - output-oriented is given by Eq. (2):

$$\min h_k = \sum_{r=1}^{n} v_r\, x_{rk} \tag{2}$$

Subject to:

$$\sum_{i=1}^{m} u_i\, y_{ik} = 1$$
$$\sum_{r=1}^{m} u_r\, y_{rj} - \sum_{i=1}^{n} v_i\, x_{ij} \le 0, \forall\, j$$
$$u_r, v_i \ge 0, \forall\ r, i$$

with:

$y = outputs$; $x = inputs$
$u, v =$ weights;
$r = 1, \ldots, m$; $i = 1, \ldots, n$;
$j = 1, \ldots, N$

2.3. BCC model

The BCC model emerged as a continuation of the CCR model, created by Banker et al. (1984, p. 1086). Unlike the CCR model, the BCC model, also known as VRS, works with Variable Returns of Scale. Whilst the efficiency index found by DEA was previously an overall value that combined scale efficiency and technical efficiency, the work of Banker et al. (1984, pp. 1088–1089) separated the effect of these overall index efficiencies from each *DMU* by decomposing the scalability effect of the total efficiency value. Therefore, the BCC model measures pure technical efficiency, linked only to administrative and technical issues as well as the efficiency of scale (Lin et al., 2009, p. 8887). Eqs. (3) and (4) represent the model for input or output orientation, respectively.

$$\max h_k = \sum_{r=1}^{m} u_r\, y_{rk} - u_k \tag{3}$$

Subject to:

$$\sum_{i=1}^{n} v_i\, x_{ik} = 1$$
$$\sum_{r=1}^{m} u_r\, y_{rj} - \sum_{i=1}^{n} v_i\, x_{ij} - u_k \le 0$$
$$u_r, v_i \ge 0$$

with:

$y = outputs$; $x = inputs$
$u, v =$ weights;
$r = 1, \ldots, m$; $i = 1, \ldots, n$;
$j = 1, \ldots, N$

BCC Model - output-oriented is given by Eq. (4).

$$\min h_k = \sum_{r=1}^{n} v_r\, x_{rk} - v_k \tag{4}$$

Subject to:

$$\sum_{i=1}^{m} v_i\, x_{ik} = 1$$
$$\sum_{r=1}^{m} u_r\, y_{rj} - \sum_{i=1}^{n} v_i\, x_{ij} - v_k \le 0$$
$$u_r, v_i \ge 0$$

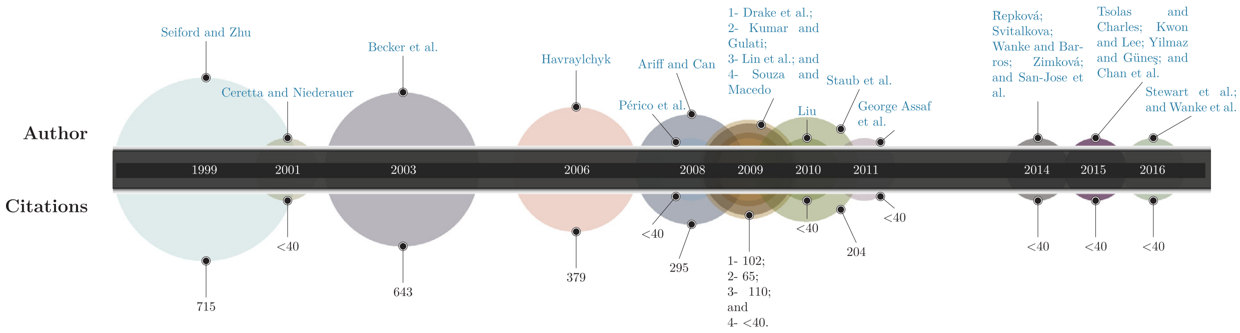

Fig. 1. Chronological order of the use of DEA for banks in the studies addressed in this paper.

with:

$y = outputs$; $x = inputs$
$u, v =$ weights;
$r = 1, \ldots, m$; $i = 1, \ldots, n$;
$j = 1, \ldots, N$

3. DEA and bank efficiency

The use of DEA to assess bank efficiency began in the work of Sherman and Gold (1985). After this study, this technique became significantly popular. It is currently one of the main techniques to measure the efficiency of the banking sector and the main non-parametric mechanism used for this purpose (Wanke et al., 2016a, p. 488).

Table 1 presents an overview of some studies that measure bank efficiency using the DEA, specifying the objectives, country of application, DEA model used, inputs and outputs and results. We included in Table 1 not only studies that analysed conventional banks but also surveys with focus on the Islamic banks, e.g., Kamarudin et al. (2015, 2016, 2017).

Fig. 1 shows the chronological order of some DEA studies measuring banking efficiency. The first study that used DEA in the banking sector, namely Sherman and Gold (1985), does not appear in chronological order due to the time lag between this study and the others. If it were included, there would be a large space in the timeline, compromising the comprehensibility of Fig. 1. In addition, Fig. 1 shows which of these articles was most cited.

Table 1, in which 28 articles are analysed, is a useful tool to visualize the main approaches used by researchers. To design this table, we selected articles that were related to the present research on DEA for banks. This was done by creating a spreadsheet containing several elements such as the objectives, the results found, the country in which DEA was applied, which variables were considered in the model and which model was used. Most of the articles were obtained using the *Elsevier ScienceDirect* search engine.

Considering the 28 articles analysed, 18 (64.28%) of the studies were applied in different countries, and 18 (64.28%) used the intermediation approach as a criterion for the selection of variables. It is important to highlight that despite the high popularity of this approach in studies involving other countries, in Brazil, from the 6 studies analysed, only (Staub et al., 2010, p. 208) and Wanke and Barros (2014) used the intermediation approach. If we do not consider the articles that analysed Brazilian banks, 72.72%, of the studies followed the intermediation, which is very expressive considering the diversity of existing methods to select inputs and outputs.[2]

Despite a large number of articles that used the financial intermediation approach, this was not a criterion for the articles to be analysed in our study. The selection of articles was only due to the relationship with the research theme. The predominance of articles that used this approach is evidence of their acceptance and academic importance. Despite this, we observe a less-frequent use of this approach in Brazilian research, evidencing the existence of one research gap.

With regard to the decomposition of efficiency and analysis of banks' RTS, Yilmaz and Güneş (2015); Seiford and Zhu (1999); Luo (2003); Kamarudin et al. (2015) addressed this issue. However, none of these surveys were carried out in Brazil, indicating an opportunity, explored in our research, to verify what the main cause of inefficiency of Brazilian banks has been, whether the problem lies in administrative and managerial skills or the scale level banks should operate at and, moreover, how to correct scale inefficiencies, indicating whether Brazilian banks should increase or reduce their scales of operation to become more efficient.

We highlight that the international literature on bank efficiency is very extensive and, therefore, the articles discussed in Table 1 depicted only a portion of the discussion. However, with regard to the publications on efficiency in the Brazilian banking sector, considering that the debate on this subject in the country is still at an initial stage, with a limited number of publications (Wanke and Barros, 2014), we sought to work with all published papers. The study allowed us to identify that international studies have been discussing topics that articles in Brazil have not yet focused on, such as the decomposition of the overall efficiency index based on the measurement of the PTE and the SE and the analysis of the RTS of the banks.

[2] All of the papers that follow the intermediation approach are marked with the symbol ⋆, in Table 1.

Table 2
Number of banks in Brazil.

	1956	1970	1987	1992	1994	1998
Federal level.	–	4	5	–	–	5
State level.	–	24	24	–	–	22
National private.	–	142	56	–	–	–
Foreign private.	–	8	18	–	–	–
Total number of banks.	**403**	**178**	**103**	**211**	**226**	**207**

Source: Adapted from Baer and Nazmi (2000, p. 6).

4. Brazilian financial system

The evolution of the Brazilian Financial System (BFS) can be divided into two major phases: before and after Law no. 4595/64. This law introduced the guidelines for the restructuring and subsequent development of the BFS, which resulted in a strong reduction in the number of banks. From Table 2, it is evident that the number of banks declined sharply after 1964. The number increased again from 1987 to 1992 due to a unique period of hyperinflation in Brazil. According to Baer and Nazmi (2000, p. 6), hyperinflation benefited the banking sector in three ways:

- It allowed loans to be collected easily, paying a negative or low real interest rate on a very high number of clients' deposits;
- It diminished the real value of banks' liabilities and reduced the likelihood of insolvency; and
- It increased liquidity by making it easier for borrowers to repay their debts.

However, in 1994, the Brazilian government launched the so-called Real Plan to control the hyperinflation. This economic plan had a major impact on the Brazilian banking sector, as it led to the end of high-spread financial transactions, which, in turn, exposed banks' inefficiencies and their difficulties in adapting to this new scenario, culminating in a relevant crisis in the sector in 1995 (Tecles and Tabak, 2010, p. 1589). In this new economic environment, the Brazilian government had to intervene through programmes such as the State Public Sector Reduction Incentive Program and the Restructuring and Strengthening Program. These measures provoked a wave of bank mergers and acquisitions, the entry of several international institutions and the overtaking of smaller banks by larger banks. This strategic movement provided for the consolidation and concentration of the BFS.

Table 3 depicts how the Brazilian banking sector has been concentrating over the years. Among the indicators used by the Brazilian Central Bank to measure concentration, RC_4 and RC_{10} are presented in Table 3. The first indicator represents the cumulative share of the four largest competitors in the market, while the second represents the cumulative share of the 10 largest competitors in the market.

From the first years of the table, it is not clear for any of the variables that the market became more concentrated. However, the concentration of the sector becomes evident when comparing the initial years with more recent years. The four largest banks held 51.47% of the BFS's total asset value, 59.80% of deposits and 59.62% of credit operations in 1995 versus 70.25%, 76.01% and 76.06% in 2014, respectively. Thus, it is possible to conclude that the participation of the four largest banks increased significantly in the analysed period. Likewise, the RC_{10} followed a similar trend as the RC_4. The 10 largest also increased their participation in the total segment. In 2014, these banks accounted for an astonishing 90% of all assets, credit operations and deposits.

5. Method

The empirical part of this paper utilizes the DEA to answer the research questions. The DEA method has excelled in academic work and for organizations that aim to evaluate their performance against competitors, as the main non-parametric technique currently used to evaluate efficiency (Wanke et al., 2016a, p. 488). Moreover, Svitalkova (2014, p. 645) points out that non-parametric techniques are better suited to ranking the efficiency of banking institutions.

Table 3
Level of concentration in the banking sector.

		1995	1999	2003	2007	2011	2014
RC_4	Total assets.	51.47	50.82	56.43	52.58	68.04	70.25
	Deposits.	59.80	57.02	61.84	59.32	72.85	76.01
	Loans.	59.62	62.89	53.79	54.55	69.91	76.06
RC_{10}	Total assets.	70.59	71.82	79.69	80.52	88.90	89.80
	Deposits.	78.22	74.48	86.39	86.18	90.06	91.51
	Loans.	82.52	80.81	78.09	84.49	90.67	91.95

5.1. DEA model and efficiency indexes

The DEA relies on several models, and defining which to use is of paramount importance for the study. Svitalkova (2014, pp. 649–651), Yilmaz and Güneş (2015, p. 387), Stewart et al. (2016, p. 103) and Seiford and Zhu (1999, p. 1274) used the CCR and BCC models as well as other variations of the DEA. Following these authors, we use the input-oriented CCR and BCC models in our study. The reason for using both models is because there is no consensus in the literature on which model is best for evaluating financial institutions. CCR should only be used if all analysed *DMUs* are operating at an optimal scale level (Řepková, 2014 p. 589, Assaf et al., 2011 p. 5782). As it is impossible to accurately state the optimal operating level in the analysed database, we use the two models to provide a more complete and global view of the banks' efficiency.

Using both the CCR and BCC models also allows us to understand the cause of inefficiency of the *DMUs*, since, when comparing the results of the models, it is possible to determine whether a *DMU* is being inefficient due to the technical part of its operation itself or if the inefficiency stems from the scale of the operational level ((Řepková, 2014) p. 594, (Yilmaz and Güneş, 2015) p. 387). The CCR model provides Technical Efficiency, also called overall efficiency, while the resulting BCC model index is known as Pure Technical Efficiency, which measures efficiency based on administrative capacity alone (Yilmaz and Güneş, 2015, p. 387). In other words, PTE is related to TE, disregarding the impact of economies or diseconomies of scales (Řepková, 2014, p. 594).

By dividing TE by PTE, it is possible to determine how efficient the analysed *DMU* is in terms of Scale Efficiency, since by adjusting the overall efficiency index-which includes both scale and purely technical-by the purely technical indicator, we obtain an index that only refers to the scale (Yilmaz and Güneş, 2015, p. 387). The scale efficiency can therefore be obtained by the ratio presented in Eq. (5).

$$SE = \frac{TE}{PTE} \tag{5}$$

Although SE measures whether DMUs are efficient or inefficient in scale, it does not provide information on how the DMU should scale its operations to become efficient (Kamarudin et al., 2015). It is necessary to evaluate the returns of scale. Graphically, on one hand, if a DMU is positioned after the most productive level of scale or the most productive scale size (MPSS),[3] it will be in the DRS zone, and therefore to become efficient, it should reduce the scale of its operations. On the other hand, if a DMU is below the MPSS, it should increase its scale, since it is having IRS. The optimum level will be reached when a DMU is obtaining constant returns to scale (CRS).

5.2. Orientation, datasets and DMUs

The model can be input-oriented, where outputs are maintained as constant and inputs are reduced to seek efficiency, or output-oriented, with the objective to keep the level of inputs constant and increase the outputs. We adopted an input orientation in this study following Schaffnit et al. (1997, p. 279), since banks generally have no control over the levels of service demanded by their customers. It is more consistent to have a bank reduce its number of employees, for example, than to increase its total loans, as this would depend on third parties' decision making.

Regarding the data used, the Brazilian Central Bank releases a quarterly database with information from the banks in the national financial system. Data such as assets, liabilities, expenses and revenues are disclosed. Based on this dataset, we selected banks from a specific category of banks with commercial portfolios. Considering that DEA is a technique of relative efficiency and that the database should be homogeneous to generate more consistent efficiency indices, the following criteria were adopted:

- Banks need to have data for the variables in all of the analysed years;
- Banks must manage personal accounts, and in addition, the services rendered to clients should not be restricted to investment portfolio management; and
- Banks that operate in very specific niches and only finance clients in certain sectors, such as the acquisition of agricultural machinery, were excluded from the analysis.

The data contained, on average, 100 banks for each year. Considering the aforementioned criteria, 37 banks were analysed. Table 4 shows which banks were studied as well as their respective values for each variable. The period of analysis began in March 2012 and ended in March 2016. The investigation of *DMUs* throughout the years is important for avoiding, or at least diminishing, the chance that an inefficient bank would be considered efficient in the study due to some short-term peculiarity (Yilmaz and Güneş, 2015, p. 389). Because the data encompass more than one year, it is unlikely that an occasional factor may bias the overall results.

The results were analysed for each year by investigating which *DMUs* were efficient and which were inefficient. Yilmaz and Güneş (2015, p. 388) argue that building a frontier for each specific year is more flexible and more appropriate than estimating a single frontier over several years. Following Yilmaz and Güneş (2015), therefore, our study was restricted to studying each year individually, without an analysis to explain if a bank or group's efficiency has been increasing or decreasing over time.

[3] For more information on how MPSS is estimated, please see Banker (1984).

Table 4
Variables considered in this paper.

Bank	DMUs	2012 – 2016			
		Fixed Assets	**Total Deposits**	**Personnel Expenses**	**Total Loans**
Alfa.	1	R$ 263.064,00	R$ 3.152.884,00	R$ 38.629,00	R$ 6.708.825,00
Bonsucesso.	2	R$ 132.354,00	R$ 1.606.829,00	R$ 5.161,00	R$ 1.709.827,00
Semear.	3	R$ 1.305,00	R$ 390.748,00	R$ 606,00	R$ 332.350,00
Topázio.	4	R$ 7.138,00	R$ 182.631,00	R$ 1.423,00	R$ 152.562,00
Banestes.	5	R$ 186.888,00	R$ 6.211.912,00	R$ 51.637,00	R$ 3.245.046,00
Banif.	6	R$ 23.576,00	R$ 1.234.144,00	R$ 17.085,00	R$ 1.205.508,00
Banrisul.	7	R$ 588.592,00	R$ 23.043.775,00	R$ 265.498,00	R$ 20.654.144,00
BB.	8	R$ 28.315.702,00	R$ 447.483.690,00	R$ 3.773.188,00	R$ 403.871.457,00
Arbi.	9	R$ 8.378,00	R$ 54.687,00	R$ 1.053,00	R$ 47.526,00
Capital.	10	R$ 920,00	R$ 6.396,00	R$ 433,00	R$ 4.840,00
Cooperativo Sicredi.	11	R$ 83.787,00	R$ 7.217.003,00	R$ 19.661,00	R$ 7.313.503,00
Banco da Amazônia.	12	R$ 230.005,00	R$ 3.326.272,00	R$ 90.667,00	R$ 2.144.069,00
Banco da China Brasil.	13	R$ 1.479,00	R$ 147.747,00	R$ 2.235,00	R$ 39.278,00
Banese.	14	R$ 73.945,00	R$ 2.303.964,00	R$ 27.366,00	R$ 1.469.866,00
Banpará.	15	R$ 45.107,00	R$ 2.439.101,00	R$ 31.516,00	R$ 1.699.206,00
BNB.	16	R$ 196.162,00	R$ 9.197.921,00	R$ 293.737,00	R$ 10.883.668,00
Fibra.	17	R$ 407.235,00	R$ 5.859.527,00	R$ 56.494,00	R$ 7.538.473,00
Ficsa.	18	R$ 2.488,00	R$ 362.497,00	R$ 2.570,00	R$ 299.086,00
La Nacion Argentina.	19	R$ 16.927,00	R$ 2.258,00	R$ 1.053,00	R$ 15.904,00
Luso Brasileiro.	20	R$ 13.539,00	R$ 358.877,00	R$ 3.932,00	R$ 248.926,00
Rep. Oriental Uruguay BCE.	21	R$ 2.778,00	R$ 485,00	R$ 349,00	R$ 2.113,00
Ribeirão Preto	22	R$ 1.175,00	R$ 78.755,00	R$ 1.185,00	R$ 221.951,00
BMG.	23	R$ 1.621.342,00	R$ 8.807.050,00	R$ 32.258,00	R$ 12.847.496,00
Bradesco.	24	R$ 37.308.373,00	R$ 214.252.701,00	R$ 2.381.620,00	R$ 239.682.243,00
BRB.	25	R$ 147.630,00	R$ 6.739.815,00	R$ 111.186,00	R$ 5.320.818,00
CEF.	26	R$ 7.592.535,00	R$268.807.536,00	R$ 3.047.989,00	R$ 268.830.701,00
Citibank.	27	R$ 1.032.053,00	R$ 15.280.070,00	R$ 326.884,00	R$ 12.946.244,00
HSBC.	28	R$ 4.231.247,00	R$ 70.636.585,00	R$ 620.277,00	R$ 47.572.461,00
Intermedium.	29	R$ 4.854,00	R$ 567.334,00	R$ 3.266,00	R$ 650.217,00
Itaú.	30	R$ 42.752.864,00	R$ 239.722.397,00	R$ 2.490.973,00	R$ 294.837.718,00
Mercantil do Brasil.	31	R$ 212.402,00	R$ 7.421.025,00	R$ 81.327,00	R$ 6.966.711,00
Original.	32	R$ 114.827,00	R$ 720.069,00	R$ 9.853,00	R$ 1.080.041,00
Panamericano.	33	R$ 167.648,00	R$ 5.761.462,00	R$ 28.358,00	R$ 5.299.324,00
Rendimento.	34	R$ 26.421,00	R$ 525.759,00	R$ 21.764,00	R$ 453.313,00
Safra.	35	R$ 2.016.581,00	R$ 15.352.876,00	R$ 243.905,00	R$ 41.246.635,00
Santander.	36	R$ 26.159.640,00	R$ 122.926.796,00	R$ 1.439.122,00	R$ 175.692.596,00
Sofisa.	37	R$ 298.051,00	R$ 2.485.194,00	R$ 15.278,00	R$ 1.820.686,00
Average		**R$ 4.169.973,30**	**R$ 40.396.453,30**	**R$ 419.987,51**	**R$ 42.839.333,30**
Alfa.	1	R$ 262.590,00	R$ 2.624.511,00	R$ 42.874,00	R$ 7.875.935,00
Bonsucesso.	2	R$ 84.891,00	R$ 1.511.788,00	R$ 12.829,00	R$ 2.368.289,00
Semear.	3	R$ 588,00	R$ 271.141,00	R$ 700,00	R$ 243.836,00
Topázio.	4	R$ 9.907,00	R$ 123.491,00	R$ 1.503,00	R$ 156.188,00
Banestes.	5	R$ 200.726,00	R$ 7.422.024,00	R$ 60.931,00	R$ 3.467.773,00
Banif.	6	R$ 10.823,00	R$ 1.000.500,00	R$ 11.277,00	R$ 1.067.483,00
Banrisul.	7	R$ 580.703,00	R$ 27.560.015,00	R$ 294.414,00	R$ 23.506.390,00
BB.	8	R$ 28.651.510,00	R$ 468.744.731,00	R$ 4.273.548,00	R$ 500.633.317,00
Arbi.	9	R$ 8.530,00	R$ 56.707,00	R$ 1.506,00	R$ 37.844,00
Capital.	10	R$ 394,00	R$ 2.888,00	R$ 494,00	R$ 5.328,00
Cooperativo Sicredi.	11	R$ 95.068,00	R$ 9.461.161,00	R$ 26.680,00	R$ 9.031.976,00
Banco da Amazônia.	12	R$ 215.481,00	R$ 2.664.960,00	R$ 105.860,00	R$ 2.056.129,00
Banco da China Brasil.	13	R$ 8.179,00	R$ 211.249,00	R$ 3.070,00	R$ 121.596,00
Banese.	14	R$ 82.444,00	R$ 2.434.054,00	R$ 31.740,00	R$ 1.607.868,00
Banpará.	15	R$ 58.888,00	R$ 3.248.183,00	R$ 36.768,00	R$ 2.292.919,00
BNB.	16	R$ 223.712,00	R$ 10.015.264,00	R$ 277.853,00	R$ 10.980.575,00
Fibra.	17	R$ 262.561,00	R$ 5.003.601,00	R$ 47.638,00	R$ 6.995.633,00
Ficsa.	18	R$ 1.945,00	R$ 269.169,00	R$ 2.205,00	R$ 347.125,00
La Nacion Argentina.	19	R$ 16.718,00	R$ 2.351,00	R$ 1.157,00	R$ 28.524,00
Luso Brasileiro.	20	R$ 8.128,00	R$ 367.427,00	R$ 3.482,00	R$ 362.007,00
Rep. Oriental Uruguay BCE.	21	R$ 2.878,00	R$ 2.043,00	R$ 415,00	R$ 7.347,00
Ribeirão Preto.	22	R$ 1.171,00	R$ 61.636,00	R$ 1.303,00	R$ 129.519,00
BMG.	23	R$ 1.654.113,00	R$ 7.655.125,00	R$ 31.683,00	R$ 19.266.320,00
Bradesco.	24	R$ 42.597.463,00	R$ 206.777.038,00	R$ 2.553.543,00	R$ 268.633.695,00
BRB.	25	R$ 167.090,00	R$ 6.676.285,00	R$ 139.886,00	R$ 6.640.402,00
CEF.	26	R$ 8.119.556,00	R$ 323.303.139,00	R$ 3.727.703,00	R$ 382.626.695,00
Citibank.	27	R$ 1.209.804,00	R$ 15.137.216,00	R$ 359.584,00	R$ 14.318.437,00
HSBC.	28	R$ 4.348.722,00	R$ 52.002.868,00	R$ 672.124,00	R$ 50.677.737,00

(*continued on next page*)

Table 4 (*continued*)

Bank	DMUs	2012 – 2016			
		Fixed Assets	**Total Deposits**	**Personnel Expenses**	**Total Loans**
Intermedium.	29	R$ 12.325,00	R$ 504.925,00	R$ 3.841,00	R$ 920.347,00
Itaú.	30	R$ 57.266.429,00	R$ 245.393.857,00	R$ 2.568.097,00	R$ 317.332.179,00
Mercantil do Brasil.	31	R$ 205.563,00	R$ 7.990.588,00	R$ 82.926,00	R$ 8.617.026,00
Original.	32	R$ 151.112,00	R$ 601.416,00	R$ 29.622,00	R$ 844.965,00
Panamericano.	33	R$ 1.233.356,00	R$ 7.053.135,00	R$ 46.371,00	R$ 10.555.278,00
Rendimento.	34	R$ 33.910,00	R$ 508.043,00	R$ 23.473,00	R$ 486.442,00
Safra.	35	R$ 2.168.817,00	R$ 10.503.450,00	R$ 225.754,00	R$ 41.871.919,00
Santander.	36	R$ 24.156.654,00	R$ 123.027.958,00	R$ 1.486.789,00	R$ 187.162.324,00
Sofisa.	37	R$ 307.986,00	R$ 2.154.437,00	R$ 16.505,00	R$ 1.523.032,00
Average		**R$ 4.714.073,92**	**R$ 41.955.361,46**	**R$ 465.031,03**	**R$ 50.940.551,32**
Alfa.	1	R$ 263.465,00	R$ 383.965,00	R$ 42.670,00	R$ 6.943.683,00
Bonsucesso.	2	R$ 60.418,00	R$ 1.384.015,00	R$ 12.299,00	R$ 1.641.418,00
Semear.	3	R$ 1.887,00	R$ 268.942,00	R$ 2.065,00	R$ 262.653,00
Topázio.	4	R$ 5.897,00	R$ 141.643,00	R$ 1.796,00	R$ 152.931,00
Banestes.	5	R$ 206.905,00	R$ 8.259.090,00	R$ 63.536,00	R$ 3.909.590,00
Banif.	6	R$7.471,00	R$ 799.621,00	R$ 10.131,00	R$ 664.111,00
Banrisul.	7	R$ 651.412,00	R$ 30.930.233,00	R$ 405.971,00	R$ 26.141.358,00
BB.	8	R$ 28.160.212,00	R$ 482.501.757,00	R$ 4.403.930,00	R$ 592.034.104,00
Arbi.	9	R$ 8.692,00	R$ 56.237,00	R$ 1.448,00	R$ 43.033,00
Capital.	10	R$ 382,00	R$ 4.105,00	R$ 573,00	R$ 6.599,00
Cooperativo Sicredi.	11	R$ 111.611,00	R$ 10.396.285,00	R$ 22.387,00	R$ 11.432.616,00
Banco da Amazônia.	12	R$ 238.618,00	R$ 3.393.178,00	R$ 101.848,00	R$ 2.597.012,00
Banco da China Brasil.	13	R$ 8.650,00	R$ 133.838,00	R$ 4.014,00	R$ 206.160,00
Banese.	14	R$ 86.187,00	R$ 2.818.618,00	R$ 32.487,00	R$ 1.623.797,00
Banpará.	15	R$ 89.598,00	R$ 4.128.474,00	R$ 44.166,00	R$ 2.798.329,00
BNB.	16	R$ 233.948,00	R$ 10.576.155,00	R$ 427.614,00	R$ 11.275.508,00
Fibra.	17	R$ 622.899,00	R$ 3.796.919,00	R$ 42.136,00	R$ 4.697.062,00
Ficsa.	18	R$ 1.685,00	R$ 204.102,00	R$ 3.936,00	R$ 114.370,00
La Nacion Argentina.	19	R$ 16.525,00	R$ 1.547,00	R$ 1.212,00	R$ 7.392,00
Luso Brasileiro.	20	R$ 3.651,00	R$ 346.789,00	R$ 3.719,00	R$ 359.510,00
Rep. Oriental Uruguay BCE.	21	R$ 2.675,00	R$ 614,00	R$ 441,00	R$ 8.090,00
Ribeirão Preto.	22	R$ 1.175,00	R$ 57.576,00	R$ 1.470,00	R$ 125.714,00
BMG.	23	R$ 1.543.768,00	R$ 6.667.383,00	R$ 48.986,00	R$ 16.943.357,00
Bradesco.	24	R$ 44.991.423,00	R$ 219.741.070,00	R$ 2.714.113,00	R$ 296.391.503,00
BRB.	25	R$ 237.536,00	R$ 7.995.076,00	R$ 156.855,00	R$ 8.232.279,00
CEF.	26	R$ 10.174.928,00	R$ 374.857.368,00	R$ 4.284.210,00	R$ 511.504.839,00
Citibank.	27	R$ 754.109,00	R$ 14.748.956,00	R$ 311.077,00	R$ 11.143.551,00
HSBC.	28	R$ 4.055.324,00	R$ 57.133.280,00	R$ 768.687,00	R$ 54.214.224,00
Intermedium.	29	R$ 11.527,00	R$ 645.803,00	R$ 7.365,00	R$ 1.157.829,00
Itaú.	30	R$ 59.751.819,00	R$ 293.130.621,00	R$ 2.668.009,00	R$ 353.406.417,00
Mercantil do Brasil.	31	R$ 261.355,00	R$ 8.337.280,00	R$ 94.109,00	R$ 8.691.221,00
Original.	32	R$ 48.535,00	R$ 384.807,00	R$ 24.805,00	R$ 1.330.527,00
Panamericano.	33	R$ 1.119.046,00	R$ 10.076.431,00	R$ 60.090,00	R$ 14.775.776,00
Rendimento.	34	R$ 40.204,00	R$ 462.299,00	R$ 27.874,00	R$ 411.266,00
Safra.	35	R$2.220.224,00	R$ 9.846.862,00	R$ 262.840,00	R$ 46.331.320,00
Santander.	36	R$ 20.921.943,00	R$ 133.480.210,00	R$ 1.449.631,00	R$ 193.561.550,00
Sofisa.	37	R$ 215.495,00	R$ 1.946.345,00	R$ 11.147,00	R$ 1.621.975,00
Average		**R$ 4.787.329,70**	**R$ 45.946.959,30**	**R$ 500.531,00**	**R$ 59.101.693,89**
Alfa.	1	R$ 282.479,00	R$ 39.502,00	R$ 396.428,00	R$ 6.922.215,00
Bonsucesso.	2	R$ 294.640,00	R$ 9.623,00	R$ 1.224.935,00	R$ 626.417,00
Semear.	3	R$ 1.788,00	R$ 2.026,00	R$ 379.176,00	R$ 372.763,00
Topázio.	4	R$ 5.025,00	R$ 2.680,00	R$ 197.518,00	R$ 160.427,00
Banestes.	5	R$ 217.734,00	R$ 71.187,00	R$ 8.822.828,00	R$ 3.868.728,00
Banif.	6	R$ 80.822,00	R$ 8.992,00	R$ 746.241,00	R$ 370.088,00
Banrisul.	7	R$ 768.718,00	R$ 381.588,00	R$ 34.859.706,00	R$ 29.222.432,00
BB.	8	R$ 31.235.067,00	R$ 4.908.139,00	R$ 468.493.467,00	R$ 658.723.696,00
Arbi.	9	R$ 9.094,00	R$ 1.528,00	R$ 77.363,00	R$ 45.199,00
Capital.	10	R$ 369,00	R$ 588,00	R$ 6.738,00	R$ 5.886,00
Cooperativo Sicredi.	11	R$ 129.294,00	R$ 24.787,00	R$ 11.744.517,00	R$ 13.610.388,00
Banco da Amazônia.	12	R$ 256.081,00	R$ 116.071,00	R$ 3.349.650,00	R$ 3.527.074,00
Banco da China Brasil.	13	R$ 7.542,00	R$ 4.336,00	R$ 313.629,00	R$ 258.182,00
Banese.	14	R$ 83.763,00	R$ 38.974,00	R$ 3.008.426,00	R$ 1.830.885,00
Banpará.	15	R$ 113.533,00	R$ 56.570,00	R$ 4.077.042,00	R$ 3.093.841,00
BNB.	16	R$ 229.497,00	R$ 355.545,00	R$ 11.384.860,00	R$ 13.025.568,00
Fibra.	17	R$ 151.051,00	R$ 39.609,00	R$ 3.140.254,00	R$ 3.432.238,00
Ficsa.	18	R$ 1.328,00	R$ 1.440,00	R$ 123.966,00	R$ 35.657,00

(*continued on next page*)

Table 4 (*continued*)

Bank	DMUs	2012 – 2016			
		Fixed Assets	**Total Deposits**	**Personnel Expenses**	**Total Loans**
La Nacion Argentina.	19	R$ 16.440,00	R$ 1.246,00	R$ 2.339,00	R$ 20.263,00
Luso Brasileiro.	20	R$ 3.270,00	R$ 3.847,00	R$ 399.295,00	R$ 468.930,00
Rep. Oriental Uruguay BCE.	21	R$ 2.489,00	R$ 498,00	R$ 652,00	R$ 13.015,00
Ribeirão Preto.	22	R$ 1.304,00	R$ 1.861,00	R$ 53.709,00	R$ 265.907,00
BMG.	23	R$ 1.996.748,00	R$ 31.230,00	R$ 5.075.562,00	R$ 7.602.628,00
Bradesco.	24	R$ 49.977.888,00	R$ 2.928.169,00	R$ 212.774.911,00	R$ 320.559.551,00
BRB.	25	R$ 362.607,00	R$165.237,00	R$ 8.898.788,00	R$ 9.256.424,00
CEF.	26	R$ 12.453.373,00	R$ 4.942.733,00	R$ 420.730.852,00	R$ 617.183.913,00
Citibank.	27	R$ 897.146,00	R$ 367.764,00	R$ 14.166.487,00	R$ 12.491.878,00
HSBC.	28	R$ 3.655.571,00	R$ 900.517,00	R$ 58.883.635,00	R$ 58.131.193,00
Intermedium.	29	R$ 8.602,00	R$ 12.657,00	R$ 897.156,00	R$ 1.639.700,00
Itaú.	30	R$ 75.646.023,00	R$ 3.417.637,00	R$ 320.872.362,00	R$ 410.964.768,00
Mercantil do Brasil.	31	R$ 259.617,00	R$ 83.813,00	R$ 8.547.548,00	R$ 8.543.271,00
Original.	32	R$ 343.688,00	R$ 25.099,00	R$ 703.115,00	R$ 2.231.921,00
Panamericano.	33	R$ 1.012.654,00	R$ 74.995,00	R$ 10.715.567,00	R$ 16.704.577,00
Rendimento.	34	R$ 40.720,00	R$ 29.870,00	R$ 611.732,00	R$ 350.128,00
Safra.	35	R$ 2.552.567,00	R$ 411.609,00	R$ 10.051.708,00	R$ 48.804.981,00
Santander.	36	R$ 18.732.949,00	R$ 1.512.133,00	R$ 140.880.796,00	R$ 222.286.195,00
Sofisa.	37	R$ 179.236,00	R$ 12.143,00	R$ 2.209.984,00	R$ 1.821.345,00
Average		**R$ 5.459.749,11**	**R$ 567.195,76**	**R$ 47.806.025,46**	**R$ 66.985.737,08**
Alfa.	1	R$ 323.417,00	R$ 40.626,00	R$ 309.151,00	R$ 6.748.462,00
Bonsucesso.	2	R$ 301.688,00	R$ 8.779,00	R$ 939.761,00	R$ 308.364,00
Semear.	3	R$ 1.689,00	R$ 3.023,00	R$ 567.958,00	R$ 400.229,00
Topázio.	4	R$ 4.005,00	R$ 3.197,00	R$ 266.165,00	R$ 147.256,00
Banestes.	5	R$ 276.560,00	R$ 79.967,00	R$ 9.310.156,00	R$ 3.473.396,00
Banif.	6	R$ 6.696,00	R$ 6.029,00	R$ 490.918,00	R$ 73.687,00
Banrisul.	7	R$ 956.272,00	R$ 401.681,00	R$ 37.793.700,00	R$ 29.808.188,00
BB.	8	R$ 31.221.063,00	R$ 5.246.319,00	R$ 455.560.520,00	R$ 667.786.191,00
Arbi.	9	R$ 8.558,00	R$ 1.544,00	R$ 70.717,00	R$ 46.319,00
Capital.	10	R$ 354,00	R$ 657,00	R$ 5.478,00	R$ 3.079,00
Cooperativo Sicredi.	11	R$ 151.596,00	R$ 26.463,00	R$ 10.362.623,00	R$ 14.442.009,00
Banco da Amazônia.	12	R$ 278.514,00	R$ 130.794,00	R$ 2.909.788,00	R$ 3.873.265,00
Banco da China Brasil.	13	R$ 6.451,00	R$ 4.777,00	R$ 294.503,00	R$ 484.293,00
Banese.	14	R$ 82.376,00	R$ 39.238,00	R$ 2.895.553,00	R$ 2.050.738,00
Banpará.	15	R$ 114.978,00	R$ 67.197,00	R$ 3.884.973,00	R$ 3.431.025,00
BNB.	16	R$ 236.206,00	R$ 426.027,00	R$ 10.352.508,00	R$ 12.678.428,00
Fibra.	17	R$ 78.659,00	R$ 23.233,00	R$ 2.173.689,00	R$ 2.479.147,00
Ficsa.	18	R$ 1.074,00	R$ 972,00	R$ 79.236,00	R$ 6.116,00
La Nacion Argentina.	19	R$ 16.351,00	R$ 1.251,00	R$ 4.433,00	R$ 29.052,00
Luso Brasileiro.	20	R$ 12.463,00	R$ 5.876,00	R$ 639.616,00	R$ 697.948,00
Rep. Oriental Uruguay BCE.	21	R$ 2.294,00	R$ 553,00	R$ 1.272,00	R$ 14.248,00
Ribeirão Preto.	22	R$ 1.575,00	R$ 1.698,00	R$ 67.483,00	R$ 373.867,00
BMG.	23	R$ 1.873.997,00	R$ 46.798,00	R$ 5.200.705,00	R$ 8.087.786,00
Bradesco.	24	R$ 51.076.723,00	R$ 3.209.178,00	R$ 189.864.277,00	R$ 317.809.283,00
BRB.	25	R$ 418.334,00	R$ 214.699,00	R$ 9.157.803,00	R$ 9.522.840,00
CEF.	26	R$ 13.153.796,00	R$ 5.018.876,00	R$ 451.018.737,00	R$ 672.513.474,00
Citibank.	27	R$ 619.525,00	R$ 296.551,00	R$ 14.677.936,00	R$ 16.009.264,00
HSBC.	28	R$ 3.099.668,00	R$ 894.990,00	R$ 55.709.668,00	R$ 55.630.103,00
Intermedium.	29	R$ 6.627,00	R$ 14.391,00	R$ 1.220.503,00	R$ 2.187.713,00
Itaú.	30	R$ 84.219.449,00	R$ 3.641.920,00	R$ 297.347.284,00	R$ 396.500.032,00
Mercantil do Brasil.	31	R$ 235.083,00	R$ 87.432,00	R$ 7.825.089,00	R$ 7.646.678,00
Original.	32	R$ 728.170,00	R$ 35.671,00	R$ 1.466.660,00	R$ 2.587.370,00
Panamericano.	33	R$ 840.450,00	R$ 87.330,00	R$ 12.960.426,00	R$ 16.230.243,00
Rendimento.	34	R$ 38.449,00	R$ 29.799,00	R$ 583.234,00	R$ 318.071,00
Safra.	35	R$ 3.099.710,00	R$ 440.788,00	R$ 9.228.824,00	R$ 38.610.052,00
Santander.	36	R$ 16.448.887,00	R$ 1.736.403,00	R$ 137.822.766,00	R$ 212.243.750,00
Sofisa.	37	R$ 83.495,00	R$ 16.278,00	R$ 2.885.708,00	R$ 1.738.000,00
Average		**R$ 5.676.356,81**	**R$ 602.459,59**	**R$ 46.917.562,73**	**R$ 67.756.485,57**

Note: Values are in thousands of Brazilian reais.
Source: BACEN (2015)

5.3. Definition of the criteria for choosing variables

Once the model, the orientation and the *DMUs* were defined, the next step was to establish which variables would be included in the model. Although this is perhaps the most important step in applying DEA, it is often neglected (Wagner and Shimshak, 2007, pp.

Table 5
Inputs and outputs considered in this paper.

Inputs:	***Output***:
Fixed assets (x_1). Total deposits (x_2). Personnel expenses (x_3).	Total loans (y_1).

57–58); (Wanke et al., 2016b, p. 380); (Zimková, 2014, p. 782); (Luo et al., 2012, p. 1119). Drake et al. (2009) argue that the main approaches in the literature for selecting variables, in the context of financial institutions, are:

- **Production**: in this approach, developed by Benston (1965), banks are primarily considered to be service providers for customers. The inputs involve physical variables such as labour, capital and materials. The outputs are generally related to the services available to customers, which may include deposits and loans; and
- **Intermediation**: this approach, proposed by Sealey and Lindley (1977), suggests that the main function of banks is to collect funds and convert them into loans and other profitable assets using physical capital and labour, i.e., the bank is mainly seen as an intermediary between surplus agents and deficit agents.

Berger and Humphrey (1997, p. 31) examined more than 130 studies that applied the efficiency frontier to banks in 21 countries, using parametric as well as non- parametric techniques. The authors identified that the production approach is more suitable for studies that evaluate agencies, whereas the intermediation approach is more recommended for evaluating banks. Fethi and Pasiouras (2010) reviewed 151 DEA studies in banks and found that the intermediation approach was more prevalent. Nevertheless, observing Table 3, it is possible to notice a lack of studies in Brazil following this approach. Considering this gap and that the essence of banking activity is intermediate transactions between surplus agents and deficit agents, we used the intermediation approach in the present work as a criterion for selecting inputs and outputs. Given the different possible variables, we show the inputs and outputs used in our study in Table 5.

The three selected inputs in our study, highlighted in Table 5, were also used by Svitalkova (2014), Havrylchyk (2006, p. 1984), Ariff and Can (2008, p. 265), Liu (2010, p. 2785)[4], Assaf et al. (2011, p. 5782) and Zimková (2014, p. 782). However, these authors took into account the number of employees instead of personnel expenses. Regarding the outputs, Havrylchyk (2006); Ariff and Can (2008); Drake et al. (2009); Kumar and Gulati (2009); Liu (2010); Staub et al. (2010); Assaf et al. (2011); Řepková (2014); Svitalkova (2014); Zimková (2014); Yilmaz and Güneş (2015) considered total loans, among other variables. Several papers that followed the intermediation approach also used total loans output, as in our study, considering that the basic activity of banks is to take deposits and to lend money to deficit agents.

The data analysis was conducted using R language and the *Benchmarking* package, which makes several DEA models available.

6. Results and discussion

Table 6 presents the main results of the *DMUs* for the five years analysed. A number was assigned for each bank, as explained in Table 4. Therefore, DMU_1 is *Alpha Bank*, and so on until DMU_{37}, which represents *Sofisa Bank*. The RTS (return to scale) column consists of the returns of each bank's scale. In cases in which the bank has to scale down its operations to become efficient, it has decreasing returns to scale (DRS). In contrast, if the bank can increase its overall efficiency through a growth in the operating level, it has increasing returns to scale (IRS). Finally, if the bank was efficient in SE, its return scale was constant (CRS).

In the first year of the analysis, four *DMUs* were efficient according to the CCR model, namely DMU_3, DMU_{19}, DMU_{22} and DMU_{23}. The average efficiency for this model was 0.536, with the lowest value of 0.134 belonging to DMU_{13}. Using the BCC model, the average efficiency was much higher, 0.754, with 15 efficient *DMUs*. DMU_{20} presented the minimum value of 0.306. It is noteworthy that, although this *DMU* had the worst performance in terms of PTE, it was efficient in when considering SE, indicating that its inefficiency is totally related to its managerial and administrative abilities. Despite the situation of DMU_{20}, the average SE was slightly lower than the average PTE, indicating that during that year, in general, banks faced more difficulties operating at the optimal level of scale than operating at optimal technical and administrative levels.

In 2013, the year with more efficient *DMUs* based on the CCR model, *DMUs* 1, 3, 18, 19, 22, 23, 29 and 35 were efficient when considering TE. The mean efficiency was 0.589, with the lowest value of 0.192 from DMU_{19}. Using the BCC model, the average efficiency was 0.746, with 15 efficient *DMUs*, as in the previous year. This year differs from the previous year regarding the average values of SE and PTE. In the previous year, the SE was slightly lower, but the situation was reversed in 2013, suggesting that the inefficiency of the Brazilian banks happened to be more associated with the technical and administrative dimensions of their operations.

The other years of the analysis followed the trend of 2013. The average SE was slightly higher than the average PTE, indicating that Brazilian banks should have focused on improving their managerial and administrative efficiency. However, there was also room for improvements to operate at optimal levels of scale. *Ribeirão Preto Bank* (DMU_{22}) was the only financial institution that was

[4] Liu (2010, p. 2785) used “funds”, including not only total deposits but also savings and other investments.

Table 6
Results for *DMUs* 1 to 20.

	2012				2013				2014				2015				2016				All Years		
DMUs	**TE**	**PTE**	**ES**	**RTS**	**TE**	**PTE**	**ES**	**RTS**	**TE**	**PTE**	**ES**	**RTS**	**TE**	**PTE**	**ES**	**RTS**	**TE**	**PTE**	**ES**	**RTS**	**TE**	**PTE**	**ES**
1	0.856	1.000	0.856	DRS	1.000	1.000	1.000	CRS	1.000	1.000	1.000	CRS	1.000	1.000	1.000	CRS	1.000	1.000	1.000	CRS	0.971	1.000	0.971
2	0.870	0.876	0.993	DRS	0.758	0.799	0.948	DRS	0.588	0.597	0.985	IRS	0.241	0.267	0.904	IRS	0.124	0.146	0.854	IRS	0.516	0.537	0.961
3	1.000	1.000	1.000	CRS	1.000	1.000	1.000	CRS	1.000	1.000	1.000	CRS	1.000	1.000	1.000	CRS	0.796	0.913	0.872	IRS	0.959	0.983	0.976
4	0.461	0.534	0.863	IRS	0.501	0.627	0.800	IRS	0.485	0.609	0.796	IRS	0.316	0.405	0.782	IRS	0.190	0.313	0.607	IRS	0.391	0.497	0.785
5	0.279	0.391	0.713	DRS	0.250	0.330	0.756	DRS	0.284	0.306	0.930	DRS	0.226	0.261	0.868	DRS	0.149	0.191	0.780	DRS	0.238	0.296	0.804
6	0.368	0.773	0.476	DRS	0.707	0.964	0.734	DRS	0.700	0.782	0.895	DRS	0.176	0.208	0.845	IRS	0.053	0.134	0.391	IRS	0.401	0.572	0.700
7	0.385	0.844	0.456	DRS	0.469	0.751	0.625	DRS	0.445	0.662	0.672	DRS	0.374	0.621	0.602	DRS	0.274	0.527	0.521	DRS	0.389	0.681	0.572
8	0.464	1.000	0.464	DRS	0.497	1.000	0.497	DRS	0.557	1.000	0.557	DRS	0.557	1.000	0.557	DRS	0.483	0.982	0.491	DRS	0.512	0.996	0.513
9	0.304	0.464	0.655	IRS	0.192	0.352	0.547	IRS	0.161	0.387	0.417	IRS	0.148	0.364	0.405	IRS	0.133	0.395	0.336	IRS	0.188	0.393	0.478
10	0.262	1.000	0.262	IRS	0.553	1.000	0.553	IRS	0.410	1.000	0.410	IRS	0.159	1.000	0.159	IRS	0.091	1.000	0.091	IRS	0.295	1.000	0.295
11	0.980	1.000	0.980	DRS	0.967	1.000	0.967	DRS	1.000	1.000	1.000	CRS	1.000	1.000	1.000	CRS	1.000	1.000	1.000	CRS	0.989	1.000	0.989
12	0.226	0.322	0.703	DRS	0.277	0.284	0.974	DRS	0.230	0.265	0.866	DRS	0.197	0.345	0.572	DRS	0.199	0.390	0.511	DRS	0.226	0.321	0.703
13	0.134	0.649	0.206	IRS	0.264	0.302	0.875	IRS	0.480	0.483	0.994	IRS	0.318	0.347	0.917	IRS	0.422	0.457	0.923	DRS	0.324	0.448	0.723
14	0.267	0.441	0.606	DRS	0.317	0.373	0.850	DRS	0.289	0.291	0.993	DRS	0.242	0.294	1.000	DRS	0.208	0.315	0.659	DRS	0.265	0.343	0.772
15	0.277	0.578	0.479	DRS	0.405	0.559	0.724	DRS	0.375	0.445	0.843	DRS	0.290	0.408	0.710	DRS	0.216	0.433	0.498	DRS	0.312	0.485	0.645
16	0.419	1.000	0.419	DRS	0.513	0.907	0.565	DRS	0.479	0.778	0.616	DRS	0.278	0.841	0.331	DRS	0.226	0.787	0.287	DRS	0.383	0.863	0.444
17	0.604	0.690	0.876	DRS	0.653	0.724	0.902	DRS	0.438	0.442	0.993	IRS	0.416	0.469	0.887	DRS	0.395	0.520	0.761	DRS	0.501	0.569	0.881
18	0.569	0.630	0.904	DRS	1.000	1.000	1.000	CRS	0.515	0.603	0.854	IRS	0.146	0.457	0.321	IRS	0.027	0.659	0.041	IRS	0.452	0.670	0.674
19	1.000	1.000	1.000	CRS	1.000	1.000	1.000	CRS	0.264	0.397	0.666	IRS	0.454	0.456	0.995	DRS	0.300	0.508	0.591	IRS	0.604	0.672	0.898
20	0.306	0.306	1.000	IRS	0.544	0.552	0.986	IRS	0.817	0.832	0.983	DRS	0.730	0.763	0.957	DRS	0.418	0.445	0.940	DRS	0.563	0.579	0.972

To be continued.
Note: TE = Technical efficiency. PTE = Pure technical efficiency. SE = Scale efficiency. RTS = Returns to scale.

Table 7
Results for *DMU* s 21 to 37.

	2012				2013				2014				2015				2016				All years		
DMUs	**TE**	**PTE**	**SE**	**RTS**	**TE**	**PTE**	**SE**	**RTS**	**TE**	**PTE**	**SE**	**RTS**	**TE**	**PTE**	**SE**	**RTS**	**TE**	**PTE**	**SE**	**RTS**	**TE**	**PTE**	**SE**
21	0.721	1.000	0.721	IRS	0.665	1.000	0.665	IRS	0.729	1.000	0.729	IRS	1.000	1.000	1.000	CRS	0.513	1.000	0.513	IRS	0.726	1.000	0.726
22	1.000	1.000	1.000	CRS	1.000	1.000	1.000	CRS	1.000	1.000	1.000	CRS	1.000	1.000	1.000	CRS	1.000	1.000	1.000	CRS	1.000	1.000	1.000
23	1.000	1.000	1.000	CRS	1.000	1.000	1.000	CRS	1.000	1.000	1.000	CRS	0.816	0.818	0.998	IRS	0.604	0.643	0.939	DRS	0.884	0.892	0.991
24	0.476	0.884	0.539	DRS	0.408	0.970	0.421	DRS	0.431	0.938	0.460	DRS	0.485	0.970	0.500	DRS	0.416	1.000	0.416	DRS	0.443	0.952	0.466
25	0.280	0.662	0.423	DRS	0.459	0.760	0.604	DRS	0.461	0.649	0.709	DRS	0.320	0.560	0.572	DRS	0.199	0.513	0.388	DRS	0.344	0.629	0.547
26	0.434	1.000	0.434	DRS	0.613	1.000	0.613	DRS	0.706	1.000	0.706	DRS	0.613	1.000	0.613	DRS	0.503	1.000	0.503	DRS	0.574	1.000	0.574
27	0.297	0.462	0.644	DRS	0.341	0.415	0.821	DRS	0.278	0.367	0.757	DRS	0.207	0.382	0.542	DRS	0.236	0.589	0.400	DRS	0.272	0.443	0.613
28	0.339	0.517	0.654	DRS	0.373	0.494	0.756	DRS	0.350	0.457	0.765	DRS	0.325	0.499	0.652	DRS	0.258	0.553	0.467	DRS	0.329	0.504	0.653
29	0.782	1.000	0.782	DRS	1.000	1.000	1.000	CRS	1.000	1.000	1.000	CRS	0.927	1.000	0.927	DRS	1.000	1.000	1.000	CRS	0.942	1.000	0.942
30	0.544	1.000	0.544	DRS	0.424	1.000	0.424	DRS	0.449	0.972	0.462	DRS	0.495	0.878	0.564	DRS	0.420	0.863	0.487	DRS	0.466	0.942	0.495
31	0.417	0.731	0.570	DRS	0.564	0.823	0.686	DRS	0.525	0.642	0.817	DRS	0.459	0.611	0.751	DRS	0.329	0.553	0.594	DRS	0.459	0.672	0.682
32	0.565	0.599	0.943	DRS	0.348	0.348	1.000	CRS	0.734	0.774	0.949	DRS	0.469	0.479	0.978	IRS	0.328	0.360	0.909	DRS	0.489	0.512	0.954
33	0.666	0.717	0.929	DRS	0.594	0.599	0.991	DRS	0.752	0.760	0.989	DRS	0.789	0.939	0.840	DRS	0.579	0.748	0.775	DRS	0.676	0.753	0.898
34	0.304	0.377	0.806	DRS	0.364	0.370	0.985	DRS	0.236	0.245	0.961	DRS	0.100	0.107	0.933	DRS	0.088	0.090	0.978	IRS	0.218	0.238	0.918
35	0.951	1.000	0.951	DRS	1.000	1.000	1.000	CRS	0.916	1.000	0.916	DRS	0.658	1.000	0.658	DRS	0.465	1.000	0.465	DRS	0.798	1.000	0.798
36	0.591	1.000	0.591	DRS	0.489	1.000	0.489	DRS	0.524	0.994	0.528	DRS	0.607	1.000	0.607	DRS	0.476	0.960	0.496	DRS	0.537	0.991	0.542
37	0.428	0.449	0.954	DRS	0.289	0.293	0.985	IRS	0.435	0.449	0.968	IRS	0.478	0.493	0.969	IRS	0.307	0.314	0.980	DRS	0.387	0.400	0.969
Minimum[a]	**0.134**	**0.306**	**0.206**	–	**0.192**	**0.284**	**0.421**	–	**0.161**	**0.245**	**0.410**	–	**0.100**	**0.107**	**0.159**	–	**0.027**	**0.090**	**0.041**	–	**0.188**	**0.238**	**0.295**
Maximum[a]	**1.000**	**1.000**	**1.000**	–	**1.000**	**1.000**	**1.000**	–	**1.000**	**1.000**	**1.000**	–	**1.000**	**1.000**	**1.000**	–	**1.000**	**1.000**	**1.000**	–	**1.000**	**1.000**	**1.000**
Average[a]	**0.536**	**0.754**	**0.713**	–	**0.589**	**0.746**	**0.804**	–	**0.569**	**0.706**	**0.816**	–	**0.487**	**0.655**	**0.754**	–	**0.390**	**0.630**	**0.634**	–	**0.514**	**0.698**	**0.744**

Note: TE = Technical efficiency. PTE = Pure technical efficiency. SE = Scale efficiency. RTS = Returns to scale.
[a] Values related to *DMU* s. (1 to 37).

Table 8
Efficiency ranking.

Γ	Bank	DMU	$\overline{M}$	Ω	Number of Times It Was Efficient		
					TE	PTE	SE
1º	Ribeirão Preto.	22	1.000	31º	5	5	5
2º	Cooperativo Sicredi.	11	0.989	11º	3	5	3
3º	Alfa.	1	0.971	15º	4	5	4
4º	Semear.	3	0.959	30º	4	4	4
5º	Intermedium.	29	0.942	24º	3	5	3
6º	BMG.	23	0.884	14º	3	3	3
7º	Safra.	35	0.798	7º	1	5	1
8º	Rep. Oriental Uruguay BCE.	21	0.726	36º	1	5	1
9º	Panamericano.	33	0.676	12º	0	0	0
10º	La Nacion Argentina.	19	0.604	33º	2	2	2
11º	CEF.	26	0.574	2º	0	5	0
12º	Luso Brasileiro.	20	0.563	28º	0	0	1
13º	Santander.	36	0.537	5º	0	3	0
14º	Bonsucesso.	2	0.516	25º	0	0	0
15º	BB.	8	0.512	1º	0	4	0
16º	Fibra.	17	0.501	19º	0	0	0
17º	Original.	32	0.489	20º	0	0	1
18º	Itaú.	30	0.466	3º	0	2	0
19º	Mercantil do Brasil.	31	0.459	17º	0	0	0
20º	Ficsa.	18	0.452	35º	1	1	1
21º	Bradesco.	24	0.443	4º	0	1	0
22º	Banif.	6	0.401	29º	0	0	0
23º	Topázio.	4	0.391	32º	0	0	0
24º	Banrisul.	7	0.389	9º	0	0	0
25º	Sofisa.	37	0.387	22º	0	0	0
26º	BNB.	16	0.383	10º	0	1	0
27º	BRB.	25	0.344	16º	0	0	0
28º	HSBC.	28	0.329	6º	0	0	0
29º	Banco da China Brasil.	13	0.324	27º	0	0	0
30º	Banpará.	15	0.312	21º	0	0	0
31º	Capital.	10	0.295	37º	0	5	0
32º	Citibank.	27	0.272	8º	0	0	0
33º	Banese.	14	0.265	23º	0	0	1
34º	Banestes.	5	0.238	13º	0	0	0
35º	Banco da Amazônia	12	0.226	18º	0	0	0
36º	Rendimento.	34	0.218	26º	0	0	0
37º	Arbi.	9	0.188	34º	0	0	0

Note: Γ represents the efficiency ranking; $\overline{M}$ represents the average efficiency; e **Ω** represents the size ranking.

globally efficient in all of the years analysed, followed by *Alfa Bank* (DMU_1) and *Semear Bank* (DMU_3), which were both efficient for four of the years. These banks may serve as benchmarks for the other banks.

Observing Table 7 and 8, the most efficient *DMU* was *Ribeirão Preto Bank*, followed by *Sicredi Cooperative Bank* and *Alfa Bank*. These banks were only the 31st, 15 th and 11th largest financial banks in the sample, respectively. The three largest banks (*BB*, *CEF* and *Itaú*) only occupied the 15th, 11th and 18th positions in the efficiency ranking and did not reach a maximum TE score in any year.

Nevertheless, it is valid to highlight the performance of these banks with regard to PTE. In this context, of the seven largest banks, two were efficient in the technical and administrative aspects of their operations each year: *CEF* (DMU_{26}, second largest bank) and *Safra* (DMU_{35}). Four banks had an average PTE very close to 1 (See Table 6), namely *BB* (DMU_8, the largest bank), *Itaú* (DMU_{30}), *Bradesco* (DMU_{24}) and *Santander* (DMU_{36}). The only bank of the seven largest to counter this trend was *HSBC* (DMU_{28}), which performed better at SE than PTE in four of the years.

Considering Table 6, the largest banks presented excellent performance when taking into account only their administrative and managerial characteristics but demonstrated strong inefficiency in scale, which indicates failures to operate at optimal levels of scale. The SE indicates whether banks are being efficient or inefficient in scale but does not indicate how the bank could correct its inefficiencies (Kamarudin et al., 2015), as discussed in Section 5.

Under this perspective, it is necessary to analyse the RTS of the banks. For DMU_8, DMU_{26}, DMU_{30}, DMU_{24}, DMU_{36}, DMU_{28}, DMU_{35}, DMU_{27}, DMU_7 e DMU_{16}, with the exception of the DMU_{35}, which presented constant returns in 2013, all banks presented decreasing returns of scale in all years analysed, indicating that for these banks to improve their efficiency ratios, there should be a reduction in their scales of operations. Considering that the Brazilian banking sector is very concentrated, with the ten largest banks accounting for about 90% of total assets, total deposits and total loans of the entire banking sector in the country,[5] consequently these banks have to operate at a scale higher than the ideal point. In contrast, *Ribeirão Preto Bank*, efficient in scale in all years, is a small bank and has clients almost exclusively from the municipality of Ribeirão Preto, in the state of São Paulo.

Regarding the ten smallest banks, DMU_{10}, DMU_{21}, DMU_{18}, DMU_{9}, DMU_{19}, DMU_{4}, DMU_{22}, DMU_{3}, DMU_{6} and DMU_{20} show a high predominance of IRS. From this sample of the ten smallest banks, the predominant IRS trend is reversed in the four largest banks, which started to present mainly CRS and DRS. Considering the five-year period analysed, the sample of the ten smallest banks generated 50 analyses of RTS, in which 29 indicated IRS, 13 indicated CRS and only 8 indicated DRS. The fact that the largest banks have, in the majority, DRS, whereas the smaller ones have IRS, is compatible with other studies in the literature. Similar results were found by Kamarudin et al. (2015); Luo (2003); Seiford and Zhu (1999); Yilmaz and Güneş (2015).

These results suggest that smaller banks should seek to improve their SE by increasing the scale of their operations. Some of these small banks have foreign headquarters and have very restricted participation in the Brazilian banking sector, while other banks in this sample are local banks from specific municipalities in Brazil. Possible solutions to improve the SE for these banks are to seek ways to gain market in the municipality in which they are installed, winning customers from the largest banks or aiming to expand operations with new branches in other regions. These banks may also seek to participate in mergers and acquisitions, which would result in an increase in the operating scale level.

Another managerial implication is the fact that, due to the input orientation of this study, the values found for efficiency suggest that inefficient banks should reduce their use of inputs, without simultaneously damaging their output. In other words, in order for inefficient *DMUs* to reach the efficient frontier, composed of the best practices of the analysed group, they should reduce their fixed assets, personnel expenses and total deposits, while keeping their total loans constant. Taking *BMG Bank* (DMU_{23}) as an example, to become efficient in 2015, the bank should have reduced its inputs by approximately 18.35%, without changing its output. The same must be done for all inefficient banks to become closer to the efficient frontier.

At first, it may seem not obvious to suggest that a bank should reduce its fixed assets or total deposits to become efficient. However, investment in assets is only justifiable if the bank expects to receive future economic returns. Since the basic activity of a bank is intermediation, maintaining a large asset structure, paying high salaries and capturing many deposits do not necessarily imply larger volume of loans for the bank. In this context, potential operating revenues generated from spreads in interest rates may not necessarily justify the maintenance of high levels of assets, deposits or personnel expenses.

Staub et al. (2010, pp. 211–212), authors who use DEA to analyse Brazilian banks while considering the intermediation approach, point out that the TE of the largest banks was not superior to the TE of the smaller ones. In fact, the authors identify values for larger banks that are even smaller than for smaller banks (0.65 *versus* 0.72) in the period from 2000 to 2007. Although not statistically significant, the results found by these authors may suggest the possible validity of the market niches hypothesis. Staub et al. (2010, pp. 206–208) suggest that smaller banks may have advantages by operating in specific niche markets in Brazil. The wave of mergers and acquisitions in the Brazilian banking sector may be explained by this theory, with the largest banks trying to increase their efficiency by buying the highly specialized minor banks to operate in these niches.

Périco et al. (2008, p. 428) analysed the 12 largest banks, in terms of total assets, using the BCC model. The authors found that the largest banks were not necessarily the most efficient, despite *Bradesco* and *Itaú* – the second- and fourth-largest Brazilian banks, respectively – obtaining the maximum score in efficiency. In contrast, *BB* and *CEF* – the first- and third-largest banks, respectively – were not considered efficient. It is important to highlight that *BB* and *CEF* are federally owned banks and that three of the smaller banks in the sample, namely *Citibank*, *Nossa Caixa* and *Safra*, were also ranked first in the efficiency rating.

Similar to the results found in this paper, when evaluating 27 banks in the Indian banking sector through a two-stage DEA model, Kumar and Gulati (2009, p. 66) identified that small banks performed a little better than the large ones (0. 8816 *versus* 0.8325, respectively). Ariff and Can (2008, p. 271) pointed out, using practically the same variables as this work and following the CCR and Tobit models with an intermediation approach, that the medium- sized banks were the most efficient. It should be noted that the two aforementioned works used total assets as a criterion of size.

To analyse the variables that impact efficiency, Wanke and Barros (2014, pp. 2341–2342) used a two-stage DEA model and separated the Brazilian banks into four groups according to their characteristics and size. Group 1 was composed of small banks, group 2 of the three largest banks, group 3 of investment and factoring banks and group 4 of state banks and large foreign banks. The results confirmed the hypothesis that the size of a bank impacts its efficiency. Seiford and Zhu (1999, pp. 1721–1273) argue that the largest banks in the United States are more efficient at generating profits. These banks showed decreasing returns of scale in marketability, defined by the authors as market value, earnings per share and return to investors, and increasing returns of scale in profitability. These results suggest that bank size may negatively affect marketability.

Stewart et al. (2016, p. 105) separated Vietnamese banks into four categories: small, medium, large and very banks. The results found by these authors suggest that large and very large banks are more efficient than banks in the other two categories, in both the CCR and the BCC models. The average efficiency of the third and fourth categories was 0.73 and 0.71, respectively, while the first two categories obtained a mean of 0.65, according to the CCR model. The difference was even greater when analysing the data from the BCC model perspective, in which the very large banks obtained efficiency indexes of 0.86, versus 0.70 for the smaller ones.

Ceretta and Niederauer (2001, pp. 17–18) found that conglomerates composed of the largest banks performed well above conglomerates comprising medium and small banks. The average efficiency of the largest banks was 0.78, while the average efficiency was 0.50 and the efficiency of the smaller banks was 0.40. Analysing the BGC matrix elaborated by the same authors (pp. 18–19), the superiority of the largest banks is evident. A possible justification for the different results from those presented in our paper is in the choice of variables. Ceretta and Niederauer (2001, p. 8) used data on the semi-annual volume of revenue, equity and debt, which are very different from the variables selected in our study.

[5] Please, review Table 3.

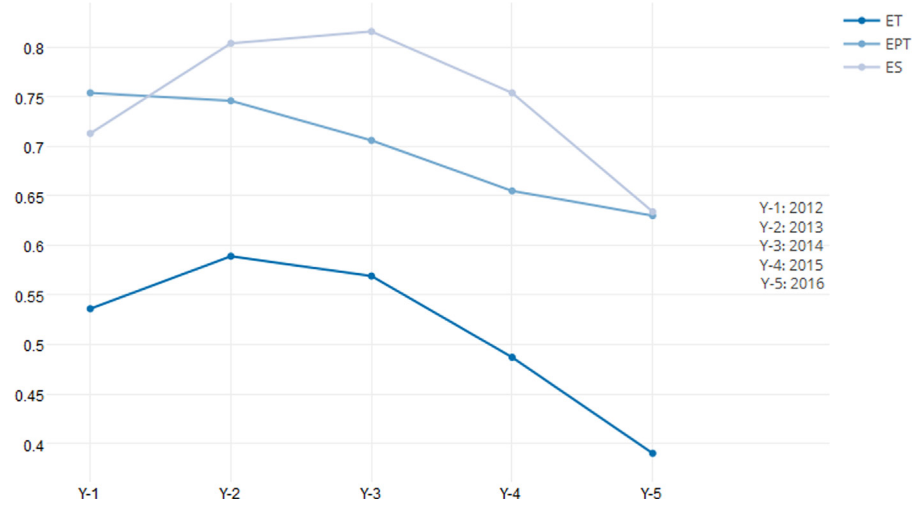

Fig. 2. Efficiency by year.

When comparing the average efficiency of the *DMUs* for each year using the CCR model and the BCC model, the first model presented much lower values for each year. The difference in the models' efficiency scores stems from the fact that the BCC model decomposes the inefficiency of the *DMUs* into purely technical efficiency and scale efficiency. Therefore, the inefficiency caused by the loss of scale is not computed in the model itself; rather, it is uncovered when comparing the results of the CCR model with those of the BCC model. While the efficiency index represents an overall result in the CCR model, it is only related to administrative and technical issues in the BCC model. However, although each model assesses efficiency with different criteria, there is clearly a strong positive correlation between them (Stewart et al., 2016, p. 104).

By analysing the studies presented earlier, it is not unanimous in the literature whether largest banks are actually more efficient. The results found in our study, with Ribeirão Preto Bank being the most efficient and the largest banks presenting problems in operating at the optimal level of scale, suggest that larger banks are not necessarily more efficient than smaller ones. This evidence is compatible with Staub et al. (2010); Ariff and Can (2008); Périco et al. (2008). However, given the diversity of the existing DEA models as well as the different approaches to variable selection, it is not possible to state that under no circumstances will the largest banks not be the most efficient, as the literature presents ambiguous results in this context.

Regarding the results presented in Fig. 2, it is not possible to state whether Brazilian banks have been improving their efficiency over the years, although the average efficiency of 2013 is higher than that of 2012. Since DEA consists of a technique of relative efficiency, all banks in 2013 may have worsened their input-output relationship; with this, the average efficiency will be higher than in the previous year, due to a possible generalized reduction in the sample performance. Similarly, it cannot be said that efficiency is worsening, although the average efficiency in 2016 was lower than that in 2015. Even with the limitations described above, the analysis of more than one year was important to find more consistent results, since, as Yilmaz and Güneş (2015, p. 389) suggest, encompassing more than one year reduces the chance of an inefficient bank being considered efficient in the study by allowing it to be analysed more than once.

7. Conclusion

The objective of this study was to analyse the efficiency of Brazilian banks, taking into account the intermediation approach and decomposing global indexes of efficiency into PTE and SE. By identifying TE, PTE and SE, one can analyse what caused a lower result for a given bank in comparison with the group. More particularly, questions related to operational inefficiencies or to administrative and technical issues can be investigated. Additionally, we analyse the returns to scale of banks, aiming to verify what procedures could be adopted to make inefficient banks in scale more efficient. Finally, we also investigated whether the largest banks would be more efficient. The period analysed was from March 2012 to March 2016. Considering a larger time span allowed us to verify whether results were consistent over the years.

This study addresses three gaps in research of banking efficiency in Brazil. Firstly, we use the intermediation approach, one of the main mechanisms used by several studies in other countries, to select the inputs and outputs of the DEA to examine the Brazilian banks in their role as financial intermediary agents. Secondly, we analyse the TE, PTE and SE and verify that inefficiency of Brazilian banks is slightly more related to technical and administrative issues than to the scale of operations, although larger banks have more opportunities for improvement in this second aspect. The five largest banks of the Brazilian financial system had excellent PTE results but were not globally efficient in any year, showing that the inefficiency of these banks was directly related to the scale level. Lastly, we study the RTS, identifying potential guidelines for inefficient banks in scale to improve their performance. In this perspective, the largest banks exhibit, on the majority, DRS, whereas smaller banks presented IRS. One of the possible causes for the failure of the largest banks to be optimal at the scale level is the need for these banks to serve a large number of clients across the country, generating a much larger asset and expense structure than smaller banks, such as Semear and Intermedium, whose average SE index was very close to 1. The smaller banks, which mainly serve specific municipalities, in contrast, need to increase their operating scale and expand to other regions of the country or participate in mergers and acquisitions to enhance their efficiency levels.

The most efficient bank was Ribeirão Preto bank, and the average efficiency of the analysed period, according to the BCC model, was 0.698 for PTE, while according to the CCR model, it was 0.514 for TE. Taking into account the high degree of inefficiency in the SE of the largest banks, which consequently affected TE, the largest banks will not necessarily be the most efficient, although it cannot be said that they can never have a good ranking in efficiency. If this work were to be restricted to applying only the BCC model, these banks would rank among the top positions in the ranking.

The results found in this study have implications for both bank managers and public agents. Bank administrators seeking to improve the efficiency of their institutions could use more efficient PTE banks as benchmarks to improve their managerial skills. In addition, considering the type of return to scale that their bank presents, they can act with the aim of expanding or reducing their scale of operations.

Results of the study may also have policy implications. For instance, the Brazilian financial sector is highly concentrated in the ten largest banks, making them operate at levels greater than ideal and compromising their efficiency indexes. Therefore, policies could be enforced with the aim to stimulate a dilution of market share in the Brazilian financial sector. Conversely, smaller banks need to expand the scope of their operations to become more efficient in scale. Combining these two factors, policies could be designed to restrict the participation of the largest banks in the financial system and stimulate the growth of smaller banks, either through mergers and acquisitions or incentives for these banks to conquer the market. With these measures, the overall efficiency of the banking sector would be enhanced.

It was not possible to state whether the efficiency of the Brazilian banking sector has improved or worsened over time, since DEA is a technique that identifies relative and not absolute efficiency. All banks may have worsened their input-output relationship, but because this is an intragroup analysis, absolute comparisons between different years are inadequate. In addition, we cannot generalize the results found by the DEA in the group analysed for all of the banks in the financial system. We recommend that future studies conduct research using appropriate techniques to analyse the change in the relative efficiency of *DMUs* over multiple periods, such as the Malmquist index. In addition, since efficiency indices may be influenced by several non-discretionary variables such as macroeconomic, sectorial and internal factors variables such as bank's control and whether the bank is public or private, future studies could use two-stage or multilevel models. For instance, in the first stage, efficiency indices could be measured by DEA models, and in the second stage, regressions techniques, i.e., truncated regression with bootstrap procedures, could be used to verify how variables would impact efficiency. In other words, these studies could investigate determinants of efficiency in Brazilian banks.

Finally, it is worth mentioning that although we intended to study all of the banks in a specific category of the Brazilian Financial System, we could not gather information on all of the variables for all of the banks. In addition, since we sought a homogeneous group of *DMUs* to get consistent results from the DEA technique, several banks had to be disregarded in the study.

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